Radio Resource Allocation in OFDMA Multihop Cellular Cooperative Networks

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Abstract—Multihop cellular networks (MCNs) is an extension of single hop cellular networks (SCNs) having benefits of a fixed base station and flexible ad hoc networks. The radio resource management becomes more complex to deal with the routing and resource allocation jointly. In this paper, we have proposed the algorithm which solves the joint subchannel allocation, routing and power allocation problem in OFDMA MCNs. Optimization problem has been decomposed into two subproblems: (1) subchannel allocation problem; (2) routing and power allocation problem, and solved by iterative two-step approach. The simulation results showed that the average total utility increases and the increment of the utility decreases as the number of hops increases.

I. INTRODUCTION

The demand for high-speed data transmission increased as the market of the wireless multimedia services grew fast in the last decade. An orthogonal frequency division multiplexing (OFDM) is one of the promising solution to increase the throughput in the system. To accommodate multiple users, the orthogonal frequency division multiple access (OFDMA) is used. Because the available resources, including time, power and subchannels, are limited for use, radio resource management is essential to utilize the limited resources efficiently in the OFDMA system. Using multihop relaying transmission, the higher throughput can be achieved than using single hop transmission. Even if the link between base station (BS) and mobile station (MS) is failed to communicate directly, the data can be delivered to the user by aid of the relay stations (RSs).

Many papers have dealt with the resource management problem in the single hop OFDMA networks. The overall transmission power is minimized in a multiuser frequency selective fading environment [1]. The total utility is maximized in a multiuser OFDM system [2][3]. The total throughput is maximized in a multi-cell environment [4]. There are, however, few papers for the resource allocation in multihop OFDMA networks. Ng [5] proposed the resource allocation scheme that not only allocates power and bandwidth for each user, but also selects the relay node and relay strategy. Bae [6] proposed the real-time heuristic algorithm for the maximization of the total capacity.

Cho and Haas [7] reported that the most of the throughput gain using multihop relaying can be obtained with the use of a two- and three-hop relaying transmission. Since they assumed noncooperative relaying scheme in the CDMA system, it is worth analyzing the performance of the multihop cooperative relaying in the OFDMA system. In addition, Ng’s and Bae’s paper considered only the two-hop relaying system excluding the three-hop relaying transmission which can increase the system throughput. In this paper, we present a resource allocation scheme which solves the joint subchannel allocation, routing and power allocation problem in OFDMA multihop cellular cooperative networks. The problem formulation of this paper is general so that multihop relaying and various relay strategies can be implemented.

The remainder of this paper is organized as follows. In section II, the system model is presented. Section III describes the iterative two-step algorithm to solve the resource allocation problem. The numerical results are shown in section IV. Finally, the conclusion is made in section V.

II. SYSTEM MODEL

A downlink OFDMA system in a multihop cellular network is considered. There are K mobile users and M mobile RSs served by a BS. The bandwidth of the system is divided into N subchannels. Let K = {0, 1, ..., M} be the set of user nodes. M = {0, 1, ..., M} denotes the set of relay nodes and 0 represents the BS. Let N = {0, 1, ..., N} be the set of subchannels. We assume that full channel state information (CSI) is available at the BS. The transmission powers of BS and RSs are limited as \[ P_{\text{max}} = [P_0, P_1, \ldots, P_M]^T \]

where \( P_m \) is the maximum power of the m-th node.

The route matrix of the k-th user on the n-th subchannel which specifies the path from the BS to the user is defined as \[ A_{k,n} = \begin{bmatrix} a_{k,n}^{(0,1)} & a_{k,n}^{(0,2)} & \cdots & a_{k,n}^{(0,M)} & a_{k,n}^{(0,M+1)} \\ a_{k,n}^{(1,1)} & a_{k,n}^{(1,2)} & \cdots & a_{k,n}^{(1,M)} & a_{k,n}^{(1,M+1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{k,n}^{(M,1)} & a_{k,n}^{(M,2)} & \cdots & a_{k,n}^{(M,M)} & a_{k,n}^{(M,M+1)} \end{bmatrix} \]

where \( a_{k,n}^{(i,j)} \) is one if the node i transmits the data to the node j when the k-th user uses the n-th subchannel, otherwise \( a_{k,n}^{(i,j)} \) is zero, and \( M + 1 \) denotes the k-th user node. The power...
allocation vector of the \( k \)-th user on the \( n \)-th subchannel is defined as \( P_{k,n} = \left[ p_{k,n,0} \quad p_{k,n,1} \quad \cdots \quad p_{k,n,M} \right] \) where \( p_{k,n,m} \) denotes the transmit power of the \( m \)-th relay node when the \( k \)-th user uses the \( n \)-th subchannel. Moreover, we denote by \( f(A_{k,n}, P_{k,n}) \) the transmission rate when the BS transmits the data to the \( k \)-th user on the \( n \)-th subchannel through the route specified by \( A_{k,n} \) using the transmission power \( P_{k,n} \).

III. RESOURCE ALLOCATION SCHEME

The radio resource allocation problem consists of three parts: (1) subchannel allocation problem; (2) routing problem; (3) power allocation problem. Subchannel allocation problem determines the subchannel which the mobile user uses. Routing problem is to determine the path along which data is transmitted from the BS to the user. Power allocation problem is to determine the power with which the node transmits the data in a time slot.

The utility function is useful to deal with heterogenous applications which have different satisfaction levels about the different services. It is assumed that the utility functions in this paper are concave and differentiable. We use the average total utility as an objective function of the optimization problem.

The optimization problem can be formulated as the following utility maximization problem,

\[
\max_{\{p_{k,n}, A_{k,n}, P_{k,n}\}} \frac{1}{K} \sum_{k=1}^{K} \sum_{n=1}^{N} U_k \left( \sum_{n=1}^{N} f(A_{k,n}, P_{k,n})p_{k,n} \right) \tag{1}
\]

subject to

\[
\sum_{k=1}^{K} \rho_{k,n} = 1, \quad \forall n \smallskip
\rho_{k,n} \in \{0, 1\}, \quad \forall k, \forall n \smallskip
\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,m} \rho_{k,n} \leq P_m, \quad \forall m \smallskip
\]

where \( \rho_{k,n} = 1 \) denotes that the \( n \)-th subchannel is allocated to the \( k \)-th user. The third constraint means that the allowable power of the \( m \)-th relay node is restricted to \( P_m \).

It is difficult to solve the problem (1) directly because some optimization variables are integer and the utility function is usually nonlinear. The Lagrangian dual optimization can be useful for this case and the duality gap is zero when the number of subchannels goes to infinity [5]. The Lagrangian for problem (1) with relaxation of the per-node power constraints is given by

\[
L(\chi, \lambda) = \frac{1}{K} \sum_{k=1}^{K} \sum_{n=1}^{N} U_k \left( \sum_{n=1}^{N} f(A_{k,n}, P_{k,n})p_{k,n} \right) \tag{2}
\]

\[-\sum_{m=0}^{M} \lambda_m \left( \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,m} \rho_{k,n} - P_m \right) \]

where \( \chi = \{(\rho_{k,n}, A_{k,n}, P_{k,n})|k \in \mathcal{K}, n \in \mathcal{N}\} \) is the set of variables which specify the resource allocation policy, \( \lambda = [\lambda_0 \quad \lambda_1 \quad \cdots \quad \lambda_M]^T \) and \( \lambda_m \) is the \( m \)-th Lagrangian dual variable. Dual objective function is given by

\[
\mathcal{L}(\lambda) = \max_{\chi} L(\chi, \lambda) \tag{3}
\]

subject to \( \sum_{k=1}^{K} \rho_{k,n} = 1, \quad \forall n \smallskip
\rho_{k,n} \in \{0, 1\}, \quad \forall k, \forall n \smallskip
\]

We can solve the problem (1) by minimizing the dual objective:

\[
\min \mathcal{L}(\lambda) \tag{4}
\]

subject to \( \lambda_m \geq 0, \quad \forall m \).

Because it is hard to obtain the differentiation of \( \mathcal{L}(\lambda) \), the subgradient method can be used to solve the minimization problem (4). The subgradient method is as follows [5][8]:

1) Initialize \( \lambda^{(0)} \).
2) Obtain \( \mathcal{L}(\lambda^{(l)}) \) at the \( l \)-th iteration.
3) Update a subgradient for \( \lambda^{(l+1)} \),

\[
\mathcal{L}(\lambda^{(l+1)}) = \mathcal{L}(\lambda^{(l)}) + v_l(\mathcal{P}^{(l)} - \mathcal{P}^{(l)}_{\max})
\]

4) Go to step 2) until convergence.

Subchannel allocation problem given power allocation vectors and route matrices

A. Subchannel Allocation

Subchannel allocation problem given \( A_{k,n}, P_{k,n}, \lambda \) for \( k = 1, ..., K \) and \( n = 1, ..., N \) can be described as the nonlinear integer programming problem,

\[
\max_{\rho} \left[ J(\rho) = \frac{1}{K} \sum_{k=1}^{K} \sum_{n=1}^{N} r_{k,n} \rho_{k,n} \right] \tag{5}
\]

subject to \( \sum_{k=1}^{K} \rho_{k,n} = 1, \quad \forall n \smallskip
\rho_{k,n} \in \{0, 1\}, \quad \forall k, \forall n \smallskip
\]

where \( r_{k,n} = f(A_{k,n}, P_{k,n}) \) and \( \rho = [\rho_{1,1}, ..., \rho_{1,N}, \rho_{2,1}, ..., \rho_{2,N}, ..., \rho_{K,1}, ..., \rho_{K,N}] \).

We follow the development of dynamic subcarrier assignment (DSA) algorithm in [3]. With the assumption that \( U_k(\cdot) \)'s are concave and differentiable, the following inequality is satisfied for all \( \rho_x \in S \),

\[
J(\rho_x) - J(\rho_y) \geq \nabla_{\rho_x} J(\rho_x)^T (\rho_x - \rho_y), \forall \rho_y \in S
\]
where \( S = \{ \rho \sum_{k=1}^{K} \rho_k, n = 1 \forall n, \text{ and } \rho_{k,n} \in \{0,1\} \forall k \forall n \} \) is the feasible region of \( \rho \). The gradient of \( J(\rho) \) is as follow

\[
\nabla_{\rho} J(\rho) = \begin{bmatrix}
\frac{1}{K} U_1'(R_1) r_{1,1} - \lambda^T p_{1,1} \\
\vdots \\
\frac{1}{K} U_k'(R_k) r_{k,1} - \lambda^T p_{k,1} \\
\vdots \\
\frac{1}{K} U_N'(R_N) r_{N,N} - \lambda^T p_{N,N}
\end{bmatrix}
\]  

(7)

where \( R_k = \sum_{n=1}^{N} r_{k,n} \rho_{k,n} \). If \( \rho^* \) satisfies

\[
\nabla_{\rho} J(\rho^*)^T (\rho^* - \rho) \geq 0, \forall \rho \in S,
\]  

(8)

then \( \rho^* \) is globally optimal. This optimal condition is equivalent to the following condition of the \( i \)-th subchannel for all \( k \neq k^*_n \),

\[
\frac{1}{K} U_k'(R_k^*) r_{k,n} - \lambda^T p_{k,n} \geq \frac{1}{K} U_k'(R_k) r_{k,n} - \lambda^T p_{k,n},
\]  

(9)

where \( R_k^* = \sum_{n=1}^{N} r_{k,n} \rho_{k,n}^* \) and \( k^*_n = \{ k | \rho_{k,n}^* = 1, k \in K \} \).

From (9) we can perform the subchannel assignment as follow

\[
k^*_n = \arg \max_{k \in K} \left[ \frac{1}{K} U_k'(R_k) r_{k,n} - \lambda^T p_{k,n} \right], \forall n.
\]  

(10)

Since \( R_k \) is also dependent on the result of subchannel assignment, the problem (5) can be solved by the following sequential algorithm [3]:

1) Initialize \( \gamma_k^{(0)} \) for all \( k \).
2) Update \( R_k^{(i+1)} \) at the \( (i+1) \)-th iteration,

\[
k_k^{(i+1)} \leftarrow \arg \max_{k \in K} \left[ \frac{1}{K} U_k^{(i)} r_{k,n} - \lambda^T p_{k,n} \right], \forall n,
\]

\[
R_k^{(i+1)} \leftarrow \sum_{n=1}^{N} r_{k,n} \rho_{k,n}^{(i+1)}, \forall k.
\]  

3) Update \( \gamma_k^{(i+1)} \) for all \( k \) as follow,

\[
\gamma_k^{(i+1)} \leftarrow (1 - \mu) \gamma_k^{(i)} + \mu U_k'(R_k^{(i+1)}), \forall k.
\]  

4) Go to step 2) until \( \frac{1}{K} \sum_{k=1}^{K} U_k'(R_k^{(i+1)}) (R_k^{(i+1)} - R_k^{(i)}) - \lambda^T (P^{(i+1)} - P^{(i)}) < \epsilon \)

where \( \mu \) is a positive step size \( \mu \in (0,1) \) and \( \epsilon \) is an arbitrary small number.

**B. Routing and Power Allocation**

Routing and power allocation problem given \( \lambda, \rho_{k,n} \) for \( k = 1, ..., K \) and \( n = 1, ..., N \) can be formulated as the following optimization problem,

\[
\max_{A_{k,n}, P_{k,n}} \frac{1}{K} \sum_{k=1}^{K} U_k \left( \sum_{n=1}^{N} f(A_{k,n}, P_{k,n}) \rho_{k,n} \right) - \sum_{n=1}^{N} \lambda^T p_{k,n}, \forall n
\]  

(11)

where \( k^*_n \) is the allocated user index for the \( n \)-th subchannel.

If the utility functions are nonlinear, the optimization problem couples across subchannel. We can, however, decouple the problem into many subproblems by linearizing the utility functions.

The first-order Taylor series approximations of the utility function about the previous rate, \( R_k^{(i)} \) is given by

\[
U_k(R_k^{(i+1)}) \approx U_k(R_k^{(i)}) + U_k'(R_k^{(i)}) (R_k^{(i+1)} - R_k^{(i)})
\]  

(12)

where \( R_k^{(i)} = \sum_{n=1}^{N} f(A_{k,n}, P_{k,n}) \rho_{k,n} \). The \( (i+1) \)-th objective function of problem (11) can be approximated by

\[
\frac{1}{K} \sum_{k=1}^{K} U_k \left( \sum_{n=1}^{N} f(A_{k,n}^{(i+1)}, P_{k,n}^{(i+1)}) \rho_{k,n} \right) - \sum_{n=1}^{N} \lambda^T p_{k,n}^{(i+1)}, \forall n
\]  

(13)

Since the first term of right-hand side from (13) is constant over \( A_{k,n}^{(i+1)} \) and \( P_{k,n}^{(i+1)} \), the optimization problem at the \( (i+1) \)-th iteration can be modified as

\[
\max_{A_{k,n}^{(i+1)}, P_{k,n}^{(i+1)}} \frac{1}{K} U_k' (R_k^{(i)}) f(A_{k,n}^{(i+1)}, P_{k,n}^{(i+1)}) - \lambda^T p_{k,n}^{(i+1)}, \forall n.
\]  

(14)

The modified optimization problem can now be decoupled into \( N \) subproblems,

\[
\max_{A_{k,n}^{(i+1)}, P_{k,n}^{(i+1)}} \frac{1}{K} U_k' (R_k^{(i)}) f(A_{k,n}^{(i+1)}, P_{k,n}^{(i+1)}) - \lambda^T p_{k,n}^{(i+1)}, \forall n.
\]  

(15)

The problem (14) can be solved iteratively by the following algorithm:

1) Initialize \( R_k^{(0)} \) for all \( k \).
2) Calculate \( U_k' (R_k^{(i)}) \) for all \( n \) at the \( i \)-th iteration.
3) For each \( n \)-th subchannel, solve the subproblem (15).
4) Go to the step 2) until convergence.
5) Update \( r_{k,n} \) for all \( k \) and \( n \) using \( A_{k,n}^{(i+1)} \) and \( p_{k,n}^{(i+1)} \).

The transmission rate function, \( f(A_{k,n}^{(i+1)}, P_{k,n}^{(i+1)}) \) can be represented as the function of the \( p_{k,n}^{(i+1)} \) in closed form given the route matrix, \( A_{k,n}^{(i+1)} \). For all possible transmission routes, \( A_{k,n}^{(i+1)} \) the maximization over power, \( p_{k,n}^{(i+1)} \) can be solved easily using the KKT conditions [9].

**C. Joint Routing, Subchannel Allocation and Power Allocation**

We have described the algorithms about subchannel allocation, routing and power allocation in the previous section III-A and III-B. The flow chart of the whole algorithm which solves the joint problem is shown in Fig. 1. The computational
TABLE I
AVERAGE TOTAL UTILITY

<table>
<thead>
<tr>
<th>Distance</th>
<th>1 hop</th>
<th>2 hop</th>
<th>3 hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 km</td>
<td>4.3321</td>
<td>4.3321</td>
<td>4.3321</td>
</tr>
<tr>
<td>1.0 km</td>
<td>3.5006</td>
<td>3.5173</td>
<td>3.5225</td>
</tr>
<tr>
<td>1.5 km</td>
<td>2.7078</td>
<td>3.1202</td>
<td>3.1413</td>
</tr>
<tr>
<td>2.0 km</td>
<td>2.0614</td>
<td>2.8754</td>
<td>2.9359</td>
</tr>
</tbody>
</table>

transmission rate functions of direct transmission, two-hop transmission and three-hop transmission are as follows [10]:

\[ f_1(A_1, p_s) = W \log(1 + \frac{\beta p_s|h_{sd}|^2}{N_0 W}), \]

\[ f_2(A_2, p_s, p_a) = \min\left\{ \frac{1}{2} W \log\left(1 + \frac{\beta p_s|h_{sa}|^2}{N_0 W}\right) + \frac{\beta p_a|h_{ab}|^2}{N_0 W}, \frac{1}{2} W \log\left(1 + \frac{\beta p_s|h_{sd}|^2}{N_0 W} + \frac{\beta p_a|h_{ab}|^2}{N_0 W}\right) \right\}, \]

\[ f_3(A_3, p_s, p_a, p_b) = \min\left\{ \frac{1}{3} W \log\left(1 + \frac{\beta p_s|h_{sd}|^2}{N_0 W}\right) + \frac{\beta p_a|h_{ab}|^2}{N_0 W}, \right. \]

\[ \frac{1}{3} W \log\left(1 + \frac{\beta p_s|h_{sd}|^2}{N_0 W} + \frac{\beta p_a|h_{ab}|^2}{N_0 W} + \frac{\beta p_b|h_{bd}|^2}{N_0 W}\right), \]

where \( h_{ij} \) denotes the channel gain from the node \( i \) to node \( j \), node \( s \) is BS, node \( a \) and \( b \) are RSs, and node \( d \) is MS. \( N_0 \) is the noise power per Hz. \( \beta \) is the capacity gap and is set to 1 in the simulation without loss of generality.

B. Simulation Results

There are 8 fixed RSs and 8 users in a cell with 2 km radius. Note that fixing the positions of RSs is just for the simulation and these RSs are wireless mobile. The 4 RSs are located in the four cardinal points, 0.75 km away from the BS. The other 4 RSs are located 1.25 km apart from the BS. The 8 users are distributed 45 degree apart. The four cases of the distances from the BS to the users are considered: 1) 0.5 km; 2) 1.0 km; 3) 1.5 km; 4) 2.0 km. Fig. 2 demonstrates the geometrical positions of the BS and RSs. The 8 users for each case are located on each dotted circle.

Table I shows the average total utility versus users’ distance from the BS with different maximum number of hops. The row of the table represents the maximum number of hops and the column of the table represents the distance between the BS and the users. In the case 1), the all users receive the data by direct communication, so they have same average total utility. As the users are far from the BS, the average total utility decreases. The average total utility of the direct transmission decreases rapidly when the users are located near cell boundary as shown in the case 4).

Fig. 3 show the total utility as the number of users increases. The locations of users are uniformly random in a cell. As the

A. Transmission Rate Functions

The cooperative relaying scheme used in the simulation is decode and forward (DF) scheme. \( f(A, p) \)'s can be obtained in closed form with the relaying scheme considered. The
impact on the increase of utility diminishes as the maximum number hops increases.

REFERENCES


From the results, we know that the three-hop relaying has less impact on the increase of the utility in comparison with two-hop relaying. However, the three-hop relaying can still increase the utility of the boundary users, and it means that the cell coverage can be expanded.

V. CONCLUSION

In this paper, we proposed the algorithm which can solve the joint subchannel allocation, routing and power allocation problem in OFDMA multihop cellular cooperative networks. We have shown that employing the three-hop relaying can increase the system utility and the coverage range of a cell. The number of users increases, the average total utility decreases because the allowable transmit powers at the BS and RS are limited. As the maximum number of hops increases, the average total utility increases and the amount of increment decreases.