Electrohydrodynamic effects on the deformation and orientation of a liquid capsule in a linear flow

Jong-Wook Ha and Seung-Man Yang

Department of Chemical Engineering, Korea Advanced Institute of Science and Technology, Taegon 305-701, Korea

(Received 26 March 1999; accepted 3 April 2000)

The role of a uniform electric field on the deformation and orientation of a liquid capsule with a viscoelastic membrane is considered analytically in the small deformation limit. The capsule is freely suspended either in a quiescent fluid or in a shear flow. The viscoelasticity of the membrane is taken into account by the Kelvin–Voigt model and the electrohydrodynamic flow is analyzed on the basis of the leaky dielectric model. In this article, we consider three different prototype models of capsules; viz., a neo-Hookean (incompressible isotropic) membrane, a red blood cell-type (area-preserving) membrane, and an interfacial-tension droplet. The deformed capsule shape from its initial sphericity and its orientaion are determined from the linearized governing equations and boundary conditions in the limit of small deformations. The asymptotic theory shows that the degree of capsule deformation induced by a uniform electric field alone is independent of the surface viscosity of the capsule as well as the viscosity ratio between the two fluids inside and outside the capsule. Meanwhile, in the presence of an imposed shear flow, the degree of deformation depends on the surface viscosity with preserving still the independence of the viscosity ratio. For an illustrative purpose, experimental results for the role of a uniform electric field on the orientation of an interfacial-tension droplet in a shear flow are discussed briefly. © 2000 American Institute of Physics.

I. INTRODUCTION

When a fluid drop is subjected to a uniform external electric field, the presence of the drop phase in turn distorts the electric field. In general, the disturbed electric field induces electric traction at the drop interface. Therefore, the external electric field can produce a variety of chemical and morphological changes in cells, which is of practical significance in various biophysical processes. Typical examples are the alignment of cells in chains as a result of electric polarization, cell deformation, induction of membrane pores and membrane fusion, and destruction of cells. Recently, Marszalek and Tsong reported a potential application of an electric field to the biological technology known as cell fission. Conventionally, heating causes protein-free lipid vesicles to deform and eventually leads to the budding of small vesicles from larger vesicles or to fragmentation of larger vesicles into smaller. However, these phenomena have been observed only at high temperatures and have not been shown to occur for cell membranes. Furthermore, this thermally induced membrane rupture cannot be controlled manually and high temperatures will probably denature cell proteins, DNA, RNA, and supramolecular structures. However, the electric-field-induced manipulation can be performed at low temperatures and has a distinct advantage for studying the mechanical strength of a membrane and for other potential applications.

The primary objective of this study is to develop the relationship between the applied electric field strength and the behavior of a cell membrane. According to Taylor, 1 when both the drop and the continuous phase are leaky dielectric fluids, the tangential as well as normal components of electric stresses are discontinuous at the interface. In order to match the imbalance in the tangential stresses, the fluids inside and outside the drop are set in motion. As a result, circulatory fluid motions inside and outside the drop are created, even in the absence of an externally imposed flow. The electric-field-induced flow outside of the drop is an axisymmetric (uniaxial or biaxial) straining flow in a qualitative manner. The drop deformation is also determined by the combined contributions from the electric stresses and the electric-field-induced viscous stresses.

Barthès-Biesel 2 considered the capsule as an incompressible Newtonian drop enclosed by an elastic or a viscoelastic membrane. It has been shown that such a particulate capsule provides an effective mathematical model for various problems of the flow-induced deformation of red blood cell (hereafter RBC), emulsions stabilized by layers of surface active agents and microcapsules produced by interfacial cross-linking polymerization in a drug delivery system. Depending on the assumptions pertaining to the physical properties of the membrane, a capsule may serve as a model for various types of particles. For example, if the membrane is supposed to be a simple liquid–liquid interface, the capsule is just an interfacial-tension drop. The membrane may be also a thin deformable solid. In the latter case, the
particle is an artificial capsule, which is commonly used in some technological applications.

Several models of capsules with an elastic membrane have been proposed. Most of them are relevant to a model of RBC in the presence of the external flow but without an imposed electric field. Barthès-Biesel and her co-workers\textsuperscript{10–12} employed an elastic membrane to consider the time-dependent deformation of an initially spherical elastic capsule in a linear shear flow in the regime of small deformations, as well as the steady-state shape of the deformed capsule. In their studies, the surface viscosity and the elastic membrane properties were also considered to obtain a better description of a RBC. In addition, the effect of finite membrane thickness was examined by Brunn.\textsuperscript{9} As one of the practical applications, the small deformation analysis can be used in determining the mechanical properties of the membrane.\textsuperscript{10–12}

In the present study, we examine the influence of the membrane on the dynamic behavior of a capsule that is immersed in an ambient linear flow under the action of a uniform dc electric field. The capsules considered here are initially spherical and filled with an incompressible Newtonian liquid. The combined contributions from the electric field and the imposed flow field are determined by means of a domain perturbation analysis for small deformation limits.\textsuperscript{13} In this procedure, we follow closely the kinematics formulation for the membrane deformation developed by Barthès-Biesel and her co-workers.\textsuperscript{4–8} However, the mechanical deforming forces in the present case are driven by both the electric stresses and the viscous stresses, which arise either from the electric-field-induced flow or from the imposed linear flow. In the following sections, the flow problems of the fluids separated by a deformable membrane are solved by satisfying the kinematic and dynamic conditions on the membrane.

II. PROBLEM STATEMENT

A. Governing equations and boundary conditions

Let us begin by considering the governing equations and boundary conditions for a capsule, which is immersed in an incompressible Newtonian fluid of viscosity $\mu$, resistivity $\chi$, and permittivity $\varepsilon$. The capsule is filled with another incompressible Newtonian fluid of viscosity $\lambda\mu$, electrical resistivity $R\chi$, and permittivity $Se\varepsilon$. As shown schematically in Fig. 1, a capsule is neutrally buoyant and immersed in an imposed linear flow under the action of a dc electric field. Otherwise, the capsule remains spherical with radius $a$. The capsule of a two-dimensional membrane separating the two fluids is characterized by a shear elastic modulus $E^t$, and a surface viscosity $\mu^s$. The membrane is assumed to be transversely isotropic and vanishingly thin. Therefore, it has no bending resistance. The dc electric field is uniform with its strength $V$ and the imposed linear flow is characterized by the undisturbed velocity field $G(E^b+\Omega)$. Here, $G$ is the strain rate and $E^b$ and $\Omega$ are the rate of strain and vorticity tensors of the imposed shear flow, respectively. The rate of strain and vorticity tensors are scaled by $G$. Nondimensionalization requires appropriate characteristic variables such as $a$ for lengths, $eV^2$ for electric and viscous stresses, and $\mu/\varepsilon V^2$ for time.

For a sufficiently small capsule, Reynolds number based on its size can be so small that inertial effects may be neglected. In addition, the electric charge convection resulted from the fluid motion is ignored. Thus, the electric field is independent of the flow field. Under these circumstances, the dynamics of the electric-field-induced flow can be described by the Stokes equations. The nondimensionalized governing equations with respect to the moving frame of reference with the origin at the center of mass of the capsule are

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{\sigma} = 0,$$

where $\mathbf{u}$ is the velocity field and the stress tensor is given as $\mathbf{\sigma} = -p\mathbf{I} + (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \mathbf{\sigma}^E$ for the fluid outside the capsule. Here, $p$ is the pressure field and $\mathbf{I}$ is the idemfactor. In addition, $\mathbf{\sigma}^E$ is the electric stress tensor that will be defined shortly. For the fluid motion inside the capsule, a similar expression can be used in terms of the properties pertaining to the fluid contained in the capsule.

Far from the capsule, the dimensionless velocity field becomes

$$\mathbf{u} \rightarrow \xi(E^b+\Omega) \mathbf{x}, \quad \text{as} \quad |\mathbf{x}| \to \infty,$$

where $\xi = \mu G/eV^2$. Furthermore, suppose that a load (or normal force per unit area) $\mathbf{f}$ is exerted by the membrane on the fluids. Then the following boundary conditions must be satisfied at the surface of the capsule designated by $\partial S$, i.e., at $\mathbf{x} \in \partial S$,

$$[\mathbf{u}]_{\partial S} = 0,$$

$$[\mathbf{\sigma} \cdot \mathbf{n}]_{\partial S} + \mathbf{f} = 0,$$
in which \( n \) is the outward unit normal to \( \partial s \). Here, the symbol \([ \cdot ]_{\partial s}\) denotes the jump of the enclosed quantity across the membrane. Thus, the conditions (3) and (4) comprise the continuity of velocities and the dynamic equilibrium at the capsule surface, respectively. The load \( f \) exerted by the membrane is related to the constitutive nature of the membrane mechanics. Finally, suppose that \( x \) is the displaced position of a point on the deformed membrane from its initial spherical surface, then the kinematic condition is given as

\[
\mathbf{u} = \frac{d\mathbf{x}}{dt} \quad (x \in \partial s).
\]  

(5)

B. Membrane mechanics

Barthès-Biesel and her co-workers\(^6,7\) formulated the membrane mechanics in the same frame of reference as the fluid problem. By doing this, both the viscous fluid stresses and the elastic solid stresses could be expressed readily in terms of Cartesian tensors. Since then, Secomb and Skalak\(^14\) considered a similar problem of a purely viscous membrane by adopting their formulation. In the present work, we also follow closely the formulation of Barthès-Biesel and Sgaier\(^7\) to describe the local viscoelastic response of a membrane deformation. First, we briefly outline their formulation to give a picture how the membrane deforms according to a viscoelastic constitutive law.

For a simple Kelvin–Voigt membrane with a strain–energy function \( W(\Lambda_1, \Lambda_2) \), the stress tensor associated with the membrane dynamics is given by the separate contributions, \( \sigma^e \) and \( \sigma^v \), from the elastic and viscous natures, respectively,

\[
\sigma^m = \sigma^e + \sigma^v = e^{-\Lambda_1} \left( \frac{\partial W}{\partial \Lambda_1} \mathbf{P} + \frac{\partial W}{\partial \Lambda_2} \Lambda \Lambda^T \right) + \mu \mathbf{P} \left[ \frac{\partial \mathbf{u}^n}{\partial x} + \left( \frac{\partial \mathbf{u}^m}{\partial x} \right)^T \right] \mathbf{P},
\]  

(6)

in which the superscript \( m \) represents the variables in the membrane phase. In this expression, \( \Lambda \) is a two-dimensional deformation gradient tensor reflecting kinematics of the membrane deformation and related to the three-dimensional deformation gradient tensor \( \mathbf{C} \), i.e., \( \Lambda = \mathbf{P} \cdot \mathbf{C} \cdot \mathbf{P} \). In addition, \( \mathbf{P} \) and \( \mathbf{P} \) denote the tensorial projections to the undeformed and deformed surfaces, respectively. Thus,

\[
\mathbf{P} = \mathbf{I} - n n^\top.
\]  

Then, the equilibrium equation that governs the membrane deformation can be written as

\[
\mathbf{P} \cdot \nabla \cdot \sigma^m = \mathbf{f}.
\]  

(7)

Tensions in the membrane are nondimensionalized by the elastic modulus \( E' \) when they are originated from the elastic nature. Otherwise, it is scaled by the electric tension \( E e V^2 / \mu \). Substituting (7) into (4) and expressing the result in terms of the scaled quantities give

\[
[\sigma \cdot n]_{\partial s} = -\kappa \mathbf{f}^e - \eta \mathbf{f}^v,
\]  

(8)

in which

\[
\mathbf{f}^e = \mathbf{P} \cdot \nabla \left[ e^{-\Lambda_1} \left( \frac{\partial W}{\partial \Lambda_1} \mathbf{P} + \frac{\partial W}{\partial \Lambda_2} \Lambda \Lambda^T \right) \right],
\]

\[
\mathbf{f}^v = \mathbf{P} \cdot \nabla \left( \mu \mathbf{P} \left[ \frac{\partial \mathbf{u}^n}{\partial x} + \left( \frac{\partial \mathbf{u}^m}{\partial x} \right)^T \right] \mathbf{P} \right).
\]

This completes a brief outline of the formulation of Barthès-Biesel and Sgaier\(^7\) for the membrane mechanics. It is worth commenting that dynamics of a Kelvin–Voigt membrane can be characterized by two independent dimensionless parameters, \( \kappa = E'/\eta a V^2 \) and \( \eta = \mu a / \mu \). The former is the ratio of elastic stresses on the membrane to electric stresses, and the latter is the viscosity ratio.

In the present study, we seek an asymptotic solution in the limit of small deformations. Thus, the displacement is asymptotically small and \( O(\sigma) \) in the limit of \( \sigma \ll 1 \), i.e.,

\[
\mathbf{x} = \mathbf{X} + O(\sigma),
\]

in which \( \mathbf{X} \) is the position vector on the undeformed surface. The small parameter \( \sigma \) will be specified shortly. The method of asymptotic expansions is straightforward if the small parameter \( \sigma \) is identified from the formulation. In the small deformation limit, \( \sigma \ll 1 \), the form of the asymptotic solution that we seek is a series expansion of \( \sigma \)-powers,\(^4\) in which the problem at each order in \( \sigma \) becomes linear.

Analogously to the problem of the capsule deformation in a shear flow, a deformation field can be expressed as

\[
\mathbf{x} = \mathbf{X} + \sigma \mathbf{K} \cdot \mathbf{x} + \sigma^2 \mathbf{X} \cdot \left( \mathbf{J} - \mathbf{K} \right) \cdot \mathbf{x} + O(\sigma^2),
\]  

(9)

in which \( \mathbf{J} \) and \( \mathbf{K} \) are symmetric and traceless, and functions of time only. The instantaneous external shape of the capsule is then

\[
r = |\mathbf{x}| = 1 + \sigma \mathbf{X} \cdot \mathbf{J} \cdot \mathbf{x} + O(\sigma^2) = 1 + \sigma \mathbf{x} \cdot \mathbf{J} \cdot \mathbf{x} / r^2 + O(\sigma^2),
\]  

(10)

with normal

\[
\mathbf{n} = \mathbf{x} / r + 2 \mathbf{x} \mathbf{x} \cdot \mathbf{J} \cdot \mathbf{x} / r^3 - 2 \mathbf{J} \cdot \mathbf{x} / r + O(\sigma^2).
\]  

(11)

In this expression, \( \mathbf{J} \) determines the overall deformation of the surface. Meanwhile, \( \mathbf{K} \) reflects in-plane deformations of the membrane elements.

C. Electric-field-induced flow

As far as a simple interfacial-tension drop is concerned, the fluids outside and inside the drop remain motionless at steady state either when the drop phase is a perfect conductor and the continuous phase is a perfect insulator or when the two contiguous fluids are perfectly dielectric materials. In these cases, the tangential components of electric stresses are always continuous at the interface, and the electrohydrostatic approach\(^15\) is strictly valid. However, Taylor\(^7\) suggested that the fluids had to be treated as dielectrics with a finite Ohmic resistance in order to describe the deformation of the drop more realistically. In his model, the fluids have small conductivities and, thus, when an electric field is applied, free charges appear at the drop interface. The charges on the two hemispheres of the drop in a uniform electric field are antisymmetric so that the net charge is zero. The action of the electric field on these charges sets the fluids in motion and
circulatory flows are induced inside and outside the droplet. Taylor’s model is able to predict both prolute and olbate deformations depending on the properties of the fluids, in qualitative agreement with the previous experiments.\textsuperscript{16–21}

Since the electric field is disturbed due to the presence of a capsule, the electric traction is exerted on the membrane. The electric stress can be expressed in terms of the Maxwell stress tensor,\textsuperscript{22} defined as

$$\sigma^E = (\nabla \phi \nabla \phi - \frac{1}{2} |\nabla \phi|^2),$$  \hspace{1cm} (12)

in which $\phi$ is the electrostatic potential. The fluids considered in this problem are neither perfect dielectrics nor perfect conducting substances so that each fluid possesses finite permittivity and electrical resistivity. In an external uniform electric field $\mathbf{V}$, the disturbed electrostatic potentials outside and inside the capsule can be written as

$$\phi_{\text{out}} = - \left(1 + \frac{m_1}{r^3}\right) \mathbf{V} \cdot \mathbf{x}, \quad \phi_{\text{in}} = - m_2 \mathbf{V} \cdot \mathbf{x},$$  \hspace{1cm} (13)

in which $m_1$ and $m_2$ are coefficients that must be determined by satisfying the appropriate boundary conditions. First, consider a simple case in which the membrane phase does not disturb the electric field. In this case, the electric fields are identical to those for the interfacial-tension droplet. The coefficients included in (13) are given by

$$m_1 = \frac{R - 1}{2R + 1} \quad \text{and} \quad m_2 = \frac{3R}{2R + 1},$$

in which $R$ denotes the ratio of the resistivity of the inner fluid to that of the outer fluid. Let us then turn to a realistic case of nontrivial electrical responses of the membrane with thickness $\Delta$. The capsule in this case can be treated as a double emulsion droplet with a thin annular phase.\textsuperscript{23} The coefficients contained in (13) are

$$m_1 = \frac{(1 + 2R_{im})(R_{om} - 1) + (1 - \Delta)^3 (R_{im} + 2)(R_{om} - 1)}{(1 + 2R_{im})(1 + 2R_{om}) + 2(1 - \Delta)^3 (1 - R_{im})(1 - R_{om})},$$

and

$$m_2 = \frac{9R_{im}R_{om}}{(1 + 2R_{im})(1 + 2R_{om}) + 2(1 - \Delta)^3 (1 - R_{im})(1 - R_{om})}.$$

In the above expressions, $R_{om}$ (or $R_{im}$) is the resistivity ratio between the fluid outside (or inside) the capsule and the membrane. Of course, when $R_{om} = 1$ or $R_{im} = 1$, the electrostatic effect induced by the membrane becomes diminished. However, the leaky dielectric model for the fluids is not entirely valid for the practical purpose, especially for the cells with a surface charge. In order to develop a more reliable model for the cell deformation in an electric field, the electrokinetic effect should be included.

In order to examine the effect of electric field on the deformation of a capsule, it is convenient to decompose the electric stresses into components normal and tangential to the interface.\textsuperscript{3,17,19} As we shall see shortly, both the normal and the tangential components of electric stresses are discontinuous across the membrane. Thus, to analyze the capsule deformation in an electric field, the flow problem coupled with the electrostatic one must be solved simultaneously. The jump of tangential components of the electric stresses across the membrane is given by

$$\frac{(1 - SR)}{R} \frac{\partial \phi_{\text{in}}}{\partial x_i} \frac{\partial \phi_{\text{in}}}{\partial x_j} n_i t_j.$$

Here, $S$ denotes the ratio of the permittivity of the inner fluid to that of the outer fluid and $t$ represents the unit tangential vector on the membrane surface. In addition, the normal component mismatch is

$$\frac{1}{2} \left( \frac{1}{R^2} (1 + R^2 - 2SR^2) \frac{\partial \phi_{\text{im}}}{\partial x_i} \frac{\partial \phi_{\text{im}}}{\partial x_j} n_i n_j - (1 - S) \frac{\partial \phi_{\text{im}}}{\partial x_i} \frac{\partial \phi_{\text{im}}}{\partial x_j} \right).$$

It is clear that both the tangential and the normal stresses are not continuous at the capsule surface except for some special cases. Consequently, the electric-field-induced flow is generated in such a way that the mismatch of tangential stresses is exactly removed.

## III. SHAPE EVOLUTION OF A CAPSULE

It is thus obvious that boundary-driven fluid motions are generated by an electric field under the assumption of the leaky dielectric model and the details of flow behavior depend on the viscoelastic nature of the membrane. Since the capsule is also subjected to an external mean flow, the overall velocity field is contributed from three different mechanisms. Due to the linearity of the Stokes equation, the overall velocity field can be obtained by superimposing the three contributions:

$$\mathbf{u} = \mathbf{u}^E + \mathbf{u}^f + \mathbf{u}^m.$$  \hspace{1cm} (14)

Here, superscripts $E$, $f$, and $m$ denote the velocity fields associated with the external electric field, the imposed flow field, and the membrane dynamics, respectively.

The electric-field-induced flow, $\mathbf{u}^E$, must satisfy the Stokes equation of motion plus continuity equation (1) and vanish at large distances from the capsule. Obviously, $\mathbf{u}^E$ should be continuous at the capsule surface. In addition, the tangential components of the electric and hydrodynamic stresses associated with the electric-field-induced flow are also continuous at the capsule surface. Under the assumption of the leaky dielectric materials, the electric-field-induced velocity at the membrane ($x \in \delta S$) can be expressed in terms of the rate-of-strain tensor $\mathbf{E}$, which is responsible for the electrohydrodynamic flow. The results are

$$\mathbf{u}^E = a_0 \mathbf{E} \cdot \mathbf{x} + a_1 \mathbf{x} \cdot \mathbf{E} \cdot \mathbf{x} + O(\sigma),$$  \hspace{1cm} (15)

where

$$E_{ij} = \delta_{ij} \frac{m}{2} \delta_{ij},$$

and

$$a_0 = \frac{(1 - m_1 - 2m_2^2) - Sm_1^2}{(3 + 2\lambda)(9m_1^2)} + \frac{(2 + 3\lambda)}{(3 + 2\lambda)(16 + 19\lambda)},$$

and

$$a_1 = \frac{9m_1^2}{(16 + 19\lambda)}.$$

Electrohydrodynamics of a liquid capsule in a linear flow

Here, $\lambda$ denotes the ratio of the viscosity of the inner fluid to that of the outer fluid. The velocity field $\mathbf{u}'$ for this problem is, in particular, for $\mathbf{x} \in \partial s$,

$$\xi^{-1} \mathbf{u}' = \Omega \cdot \mathbf{x} + \frac{5}{2\lambda + 3} E^h \cdot \mathbf{x} + O(\omega).$$  

(16)

The parameter $\xi = \mu G / \epsilon V^2$ represents the magnitude of electric stress relative to viscous stresses associated with the imposed flow. In fact, this is mathematically identical to the ratio of the electrical capillary number ($\epsilon = a e V^2 / \epsilon'$) to the capillary number ($Ca = a \mu G / E'$) for the capsule. The fast velocity field $\mathbf{u}^m$ in (14) is caused by two independent membrane characteristics. One is a purely elastic nature of the membrane and the other is associated with the viscous behavior of the membrane. For a viscoelastic membrane, the elastic contribution is simply superimposed to the viscous one in view of linearity. The velocity field produced by the purely elastic membrane or by the purely viscous membrane has been reported in the literature.5,24 The results are

$$\mathbf{u}^m = \kappa \omega (b_0 L \cdot \mathbf{x} + b_1 M \cdot \mathbf{x} + b_2 x x \cdot \mathbf{M} \cdot \mathbf{x})$$

$$+ \frac{\eta \omega}{c_0} \left( \frac{2}{c_0} (c_3 J + c_4 \mathbf{K}) \cdot \mathbf{x} \left( \frac{5 c_3}{c_0} + 2 b_0 \right) x \cdot \mathbf{J} \cdot \mathbf{x} + \left( \frac{5 c_4}{c_0} - 9 b_0 \right) x \cdot \mathbf{K} \cdot \mathbf{x} + O(\kappa \omega^2, \eta \omega^2) \right),$$  

(17)

in which

$$b_0 = 1 / (2 \lambda + 3); \quad b_1 = 2 (3 \lambda + 2) / (19 \lambda + 16) (2 \lambda + 3);$$

$$b_2 = 2 / (19 \lambda + 16),$$

and

$$c_0 = (2 \lambda + 3) (19 \lambda + 16); \quad c_3 = -2 (7 \lambda + 8);$$

$$c_4 = 47 \lambda + 48.$$

In addition, $\mathbf{J}$ and $\mathbf{K}$ denote the Jaumann derivatives of the second-order tensors $\mathbf{J}$ and $\mathbf{K}$, respectively. The other two tensors $\mathbf{L}$ and $\mathbf{M}$, which are both symmetric and traceless, are given by

$$\mathbf{L} = 4 (a_2 + a_3) \mathbf{J} - (6 a_2 + 10 a_3) \mathbf{K},$$  

(18)

$$\mathbf{M} = -4 (a_1 + 2 a_2 + 2 a_3) \mathbf{J} + (12 a_2 + 16 a_3) \mathbf{K}.$$  

(19)

The above relations are obvious since the load vector $\mathbf{f}$ in (4) requires

$$\mathbf{f} = -2 \alpha_1 \mathbf{n} + \sigma \mathbf{L} \cdot \mathbf{x} + \sigma \mathbf{xx} \cdot \mathbf{M} \cdot \mathbf{x} + O(\omega^2).$$  

(20)

The coefficients $\alpha_i$ depend on the specific elastic properties of the material, whose strain–energy function $W$ is expanded as follows:7

$$W = W_0 + \alpha_1 \Lambda^1 + \frac{1}{2} (\alpha_1 + \alpha_2) \Lambda^2 + \alpha_3 (\Lambda - \Lambda_1) + O(\omega^3).$$

It is noteworthy that for the fluid velocity corrected up to $O(\omega)$, the boundary conditions can be applied at the undeformed spherical surface rather than the actual deformed surface. This can be readily confirmed by a standard domain perturbation procedure.19 By the same token, the load $\mathbf{f}$ can be assumed to act on the unperturbed spherical surface $r = 1$ for the present purpose.

Then, a straightforward manipulation transforms the kinematic condition (5) into the time evolution of $\mathbf{J}$ and $\mathbf{K}$ as follows:

$$\omega \dot{\mathbf{J}} = \frac{5 \xi}{2 \lambda + 3} \mathbf{E}^h + (a_0 + a_1) \mathbf{E} + \kappa \omega [b_0 \mathbf{L} + (b_1 + b_2) \mathbf{M}]$$

$$- \frac{2 \eta \omega}{c_0} \left( c_1 \dot{\mathbf{J}} + c_2 \dot{\mathbf{K}} \right) + O(\kappa \omega^2, \eta \omega^2),$$  

(21)

$$\omega \dot{\mathbf{K}} = \frac{5 \xi}{2 \lambda + 3} \mathbf{E}^h + a_0 \mathbf{E} + \kappa \omega [b_0 \mathbf{L} + b_1 \mathbf{M}]$$

$$- \frac{2 \eta \omega}{c_0} \left( c_3 \dot{\mathbf{J}} + c_4 \dot{\mathbf{K}} \right) + O(\kappa \omega^2, \eta \omega^2),$$  

(22)

where

$$c_1 = 2 (\lambda + 4), \quad c_2 = 15 \lambda.$$  

Unlike the usual analysis for the electrohydrodynamic deformation of the drop suspended in a quiescent fluid, it is expected that there may occur a rotational motion with the vorticity associated with the type of external flow field. By the conventional definition of Jaumann derivative for any second-order tensor,

$$\dot{\mathbf{J}} = \frac{\partial \mathbf{J}}{\partial t} - \omega \cdot \mathbf{J} + \mathbf{J} \cdot \omega,$$  

(23)

where $\omega$ denotes the rotation tensor on the membrane. It goes to prove that $\omega$ is equal to the rotation tensor of the imposed flow at leading order, that is, $\omega = \Omega + O(\omega)$. In general, it follows that $\mathbf{J}$ and $\mathbf{K}$ are not scalar multiples of $\mathbf{E}$ or $\mathbf{E}^h$ because of the vorticity in the mean flow field. As a consequence, the capsule is expected to orient itself at an angle with respect to the principal direction of the flow and electric fields.

As previously mentioned, the problem formulated above is based on the asymptotic condition, $\omega \approx 1$ so that the degree of capsule deformation remains at $O(\omega)$. The asymptotic expansion procedures are straightforward if the small parameter $\omega$ is identified from the underlying physics. It can be expected that the capsule deforms slightly from its spherical shape if the deforming forces arising from the electric-field-induced flow is relatively weak compared to the restoring elastic forces of the membrane. The ratio of the two competing forces is $\kappa = E^h / a e V^2$. Thus, this small deformation condition corresponds to the asymptotic case of $\kappa > 1$ with $\eta = O(1)$. In this case, the small parameter $\omega$ is obviously $\omega = \kappa^{-1}$. The elastic forces are very similar to the interfacial tension forces of a fluid drop. For a deformable capsule, however, the surface viscosity puts another resistance to the deforming forces. Consequently, two more distinctively different asymptotic cases are available for the small deformation approximation, see Barthès-Biesel and Sgaard.7 One is for a very viscous membrane with relatively weak elasticity, i.e., $\eta \approx 1$ with $\kappa = O(1)$ and, thus, $\omega = \eta^{-1}$; and the other
for a very viscous membrane experiencing a strong elastic forces or a weak flow, i.e., $\kappa \approx 1$ and $\eta \approx 1$. In the latter case, either $\bar{\sigma} = \kappa^{-1}$ or $\bar{\sigma} = \eta^{-1}$.

Since the two independent dimensionless parameters, $\kappa$ and $\eta$, govern the degree of capsule deformation, a natural choice of a decisive parameter is their ratio, that is,

$$\beta = \frac{\eta}{\kappa} = \frac{\mu^* eV^2}{\mu E^2}. \quad (24)$$

The parameter $\beta$ can be regarded as the ratio of the characteristic relaxation time of the membrane $\mu^*/E^2$ to the flow time scale $\mu/eV^2$ induced by the electric field. Following Barthes-Biesel and Sgaier,$^7$ the small parameter can be defined in terms of $\beta$ when $\beta$ is $O(1)$ or less:

$$\bar{\sigma} = \frac{\kappa^{-1}}{1 + \beta}. \quad (25)$$

At this point, it should be noted that for the asymptotic conditions to be valid, $\kappa$ must be very large. Under these conditions, the time evolution equations (21) and (22), can be simplified as

$$\beta \dot{\mathbf{j}} = 5(1 + \beta) \xi \mathbf{E}^0 + A_1 \mathbf{E} + \mathbf{L} + \frac{1}{e} \mathbf{M}, \quad (26)$$

$$\beta \dot{\mathbf{K}} = \frac{1}{2}(1 + \beta) \xi \mathbf{E}^0 + \frac{1}{2} A_2 \mathbf{E} + \frac{1}{2} \mathbf{L} + \frac{1}{2} \mathbf{M}, \quad (27)$$

in which

$$A_1 = (1 + \beta)(1 - m_1 + 13m_2^2/16 - Sm_2^2)$$

and

$$A_2 = (1 + \beta)(1 - m_1 + 7m_2^2 - Sm_2^2).$$

Given viscoelastic properties of the membrane, the time evolution equations describe the asymptotic response of the capsule shape to any imposed linear flow field designated by $\mathbf{E}^0$ as well as a uniform dc electric field. A few specific problems will be discussed in the following sections.

### IV. DEFORMATION AND ORIENTATION OF AN INTERFACIAL-TENSION DROP

Let us then begin with a simple case of an interfacial-tension drop for an illustrative purpose. In this case, the membrane surface energy is proportional to its area with the interfacial tension $\gamma$ as the proportionality constant. Therefore, $\alpha_1 = \gamma$, $\alpha_2 = \alpha_3 = 0$. Accordingly, the dimensionless parameter $\kappa$ should be redefined in terms of the interfacial tension instead of the elastic modulus. The velocity field $\mathbf{u}^*\mathbf{u}$ originated from the viscoelastic properties of the membrane is irrelevant to this problem. In addition, it should be noted that the contribution from $\mathbf{K}$ does not appear in (9) for an interfacial-tension drop. In this case, apart from the trivial vorticity contribution, the kinematic condition (5) gives

$$\frac{\partial \mathbf{J}}{\partial t} = \frac{5}{2\lambda + 3} \xi \mathbf{E}^0 + (a_0 + a_1) \mathbf{E} - \frac{40(1 + \lambda)}{(2\lambda + 3)(19\lambda + 16)} \mathbf{J}. \quad (28)$$

Thus, at steady state,

$$\mathbf{J} = \frac{16 + 19\lambda}{8(1 + \lambda)} \xi \mathbf{E}^0 + \frac{(2\lambda + 3)(19\lambda + 16)}{40(1 + \lambda)} (a_0 + a_1) \mathbf{E}. \quad (29)$$

Obviously, when $\xi = 0$, the contribution from the imposed flow to the deformation or orientation of the interfacial-tension drop vanishes and the solution reduces to Taylor’s solution$^2$ for the drop deformation in an electric field alone. On the other hand, when $\xi$ is much larger than unity, the second term on the right-hand side is negligible. Consequently, the well-known solution for the drop deformation and orientation in a linear flow field is reproduced. When $\xi$ is $O(1)$, all the diagonal components and 12 and 21 components of $\mathbf{J}$ are nonzero.

In order to show a reliability of the present solutions, we carried out a series of simple experiments for orientation and deformation of an interfacial-tension drop under the simultaneous actions of an electric and a shear flow fields. The behavior of a single isolated drop was observed in a coaxial cylindrical Couette cell. The electric field was generated by a dc power supply (Glassman High Voltage Inc., Series EH). The controlled volume of fluid consisting of the drop phase was dispensed by a micropipette. The radius of the undeformed drop was maintained smaller than 0.5 mm. The deformation and orientation of the drop were observed by a CCD camera equipped with microscopic lenses. In order to examine the distinctive responses between the conducting and leaky dielectric drops, deionized water, and castor oil were used as a drop phase when the continuous phase was less conductive silicone oil. Two kinds of silicone oils with the viscosity of 9.8 and 0.98 Pa s, which were designated by SA and SD, respectively, were used. The deformation and orientation of drop immersed in a more conducting phase were also studied. In this case, silicone oil ($\mu = 0.10$ Pa s, designated by SO 100) and castor oil were used as the drop and the continuous phase, respectively.

It is well known that, in a uniform shear flow, a single Newtonian fluid drop is oriented at 45° with respect to the flow direction at the leading order and the orientation angle decreases when the higher-order correction is included.$^2$ On the other hand, the axis of symmetry of the deformed drop shape is aligned parallel to the electric field for either the prolate or the oblate deformation in the absence of the imposed flow. In the present experimental configuration, the flow field is perpendicular to the electric field, as schematically illustrated in Fig. 1. Consequently, when the drop is subjected to the electric field and shear flow simultaneously, the orientation angle of the major axis of the drop varies from 0° to 45° relative to the flow direction when the continuous phase is more conducting, i.e., when the oblate-type deformation is induced. Meanwhile, the orientation angle lies from 45° to 90° when the drop phase is much more conducting, i.e., when the prolate-type deformation is generated. If the shear flow is dominant, the orientation angle $\varphi$ approaches 45° for both cases at leading order.

The effect of electric field on the drop orientation was investigated within the region in which the orientation angle was not deviated considerably from 45° in a shear flow alone. In Fig. 2, the orientation angle calculated from (29) is
illustrated as a function of $\xi$. Also included are the experimentally determined orientation angles. Considering the inherent assumption for the present analytical procedure, the agreement with the theoretical prediction and the experimental results is quite good. When the electric field is dominant, the orientation of the drop approaches 0° if the drop phase is less conducting. On the other hand, when the drop phase is more conducting, the drop is aligned to the direction of electric field i.e., $\varphi \approx 90°$. In this case, the difference in the orientational behavior between the conducting drop and the leaky dielectric drop is not considerable. It can be found from the theoretical predictions that gradual changes in the orientation angle occur when $\xi$ increases from $\xi=0.1$ to $\xi=10$. In Figs. 3 and 4, the deformed drop shapes are reproduced for various electrical capillary numbers, $\zeta = a e V^2/E^\ast$. The discrepancy between theory and experiment is large for the oblate deformation, i.e., for the case of a less conducting silicone oil drop suspended in castor oil. This is because of the so-called electrorotation. When the continuous phase is more conducting than the drop phase, the relaxation time of the dispersed phase is larger than that of the continuous phase. In this case, a reverse dipole is generated on the drop. Then the drop is unstable to small rotational perturbations and will continue to rotate in a dc electric field even in the absence of an external flow field. The electrorotation can suppress the deformation perpendicular to the applied field direction. This is probably due to the fact that the rotation of the droplet interferes with the electrohydrodynamic flow along the interface of the drop and continuous phases.26,27

V. DEFORMATION AND ORIENTATION OF A CAPSULE

A. In a quiescent fluid

Now, consider the influence of the membrane on the overall behavior of a capsule in a uniform electric field. At first, we assume that there is no external mean flow field ($\mathbf{E} = 0$ and $\Omega = 0$), and, consequently, the fluids are quiescent in the absence of an electric field. The mechanical deforming forces are due to the electric stresses and the viscous stresses exerted by the electric-field-induced and membrane-originated flows. The electric-field-induced flow is irrotational and axisymmetric, i.e., either uniaxially or biaxially straining at the leading order. Consequently, at leading order, Jaumann derivatives may be replaced by the local time derivative $\partial/\partial t$. It is thus obvious from the absence of the vorticity that $J$ and $K$ are scalar multiples of $E$ and the axis of symmetry of the deformed drop is aligned always to the electric field. Under these circumstances, both of the second-order tensors, $J$ and $K$, are diagonal, and it follows that

$$J_{11} = -\frac{1}{2} J_{22} = J_{33} \quad \text{and} \quad K_{11} = -\frac{1}{2} K_{22} = K_{33}. \quad (30)$$

The equation of the membrane profile is obtained from (10), and may be written as
\[ r = 1 + \omega [J_{11}(x_1^2 + x_2^2 - 2x_3^2)] + O(\omega^2). \]  
(31)

In the absence of the external mean flow, the deformed capsule shape is axisymmetric with the axis of symmetry parallel to the electric field. In this case, the degree of capsule deformation can be defined conventionally as

\[ D = \frac{L - B}{L + B}, \]  
(32)

where \( L \) and \( B \) are the axis lengths of the capsule parallel and perpendicular to the direction of the electric field, respectively.

Let us consider first the neo-Hookean (three dimensionally isotropic incompressible) membrane that can be specified by the values of coefficients \( \alpha_i \) as

\[ \alpha_1 = 0, \quad \alpha_2 = 2\alpha_3 = \frac{3}{7}. \]

In this case, the characteristic surface modulus \( E' \) can be considered as the bulk Young modulus multiplied by the thickness of the sheet. Then, utilizing \( \mathbf{L} \) and \( \mathbf{M} \) given by (18) and (19), the evolution equations (26) and (27) become

\[ \beta \frac{\partial J_{11}}{\partial t} = -J_{11} + \frac{1}{3} A_1, \]

\[ \beta \frac{\partial K_{11}}{\partial t} = - \frac{1}{6} K_{11} - \frac{1}{6} A_2. \]

Then, the profile of the capsule deformation is

\[ J_{11} = -\frac{1}{7} (2A_1 + 3A_2) + z_1 e^{-\eta_1^2} + z_2 e^{-\eta_2^2}, \]  
(33)

in which \( z_1 \) and \( z_2 \) are constants specified by the initial conditions. It can be noted that the dynamic responses of the capsule shape decay exponentially on two time scales \( \beta \) and \( 3\beta \). One is associated with the pure shear mode of deformation and the other with the area dilution, as identified by Bartheès-Biesel and Sgaier.\(^7\)

The other model considered here is the RBC-type (area-preserving) membrane. Specifically, the pronounced elastic properties put a very strong resistance to deformation, in spite of a relatively low shear modulus. This type of membrane can be specified by the fact that the variation of \( \Lambda_1 \) remains at \( O(\omega) \) for the order of accuracy considered here. We can interpret the local change of the surface area in terms of the invariant \( \Lambda_1 \). According to the membrane mechanics reformulated by Bartheès-Biesel and Rallison,\(^6\)

\[ \Lambda_1 = \omega \mathbf{x} \cdot (2J - 3K) \mathbf{x} + O(\omega^2) = 0, \quad \text{or} \quad \mathbf{J} = \frac{1}{2} \mathbf{K} + O(\omega). \]  
(34)

By utilizing this relation, a single differential equation for \( \mathbf{J} \) is obtained from (26) and (27). That is,

\[ \beta \frac{\partial \mathbf{J}}{\partial t} = \frac{3}{4} A_2 \mathbf{E} - \frac{1}{2} \mathbf{J}. \]  
(35)

Thus, the capsule has only one relaxation time, equal to \( 2\beta \). The solution is

\[ J_{11} = -\frac{1}{7} A_2 + z_1 e^{-\eta_2^2}, \]  
(36)

in which \( z_1 \) is again determined from an initial condition.

Up to now, we have shown how the electrohydrodynamic theory can be applied to the problems involving a viscoelastic membrane such as a neo-Hookean or RBC-type membrane. In Figs. 5(a) and 5(b), the degree of steady deformation \( D \) in a uniform electric field is plotted as a function of \( \kappa^{-1} \) for the neo-Hookean and RBC-type membranes.

As noted, the degree of deformation of a neo-Hookean membrane is slightly larger than that of an RBC-type membrane,
which is independent of the type of deformation. According to (33) and (36), the degree of deformation of the capsule increases monotonously with $k^{-1}$. This is, of course, a consequence of the asymptotic expansion procedure. In order to consider a more realistic deformation of the capsule, the numerical procedure must be utilized. This is also true of a capsule that is not spherical initially or not axisymmetric such as a RBC of a biconcave disk in its natural shape. However, the advantage of analytical procedure for the simple initial spherical geometry is that it is able to provide fairly straightforward solutions, in which all the features of the membrane behavior can be taken into account. In addition, the main physical parameters can be identified readily by the present analytical solution.

It is interesting to note that the degree of deformation is independent of the viscosity ratio, $\lambda$, as well as the surface viscosity (or $\eta$), when the capsule is subjected to a uniform electric field alone in the absence of an imposed shear flow. In this problem, the electric-field-induced flow is axisymmetric and thus the membrane velocity is zero everywhere in the absence of an external flow field. Consequently, the steady deformation of the capsule does not depend on the viscosity ratio, $\lambda$, because the fluid inside the capsule remains motionless. Likewise, the surface viscosity (thus $\eta$) has no effect on the capsule deformation since the surface velocity of the membrane is zero under the action of a uniform electric field. As we shall see shortly, however, when the viscoelastic capsule is immersed in a shear flow, the deformation is limited if the viscous load is increased. This is due to the role of the membrane viscosity, which inhibits the continuous flow on the interface during the so-called tank-treading motion. In fact, the tank-treading motion is induced by the rotational flow. No-slip conditions at the membrane cause it to move with the local velocity field. Since the membrane is flexible and encloses a fluid, the capsule does not necessarily tumble as a rigid body. The membrane that is itself a closed stream surface at a steady state rotates continuously without experiencing a shape change. In the present case of a quiescent fluid, the rotational flow field does not exist and the boundedness of deformation by the membrane viscosity is not observed.

### B. In a linear shear flow

As shown in the previous section, since the electric-field-induced flow is irrotational and purely straining at the leading order, the axis of symmetry of the capsule is always parallel to the direction of the externally applied electric field. Meanwhile, when the purely elastic capsule is suspended in a simple shear flow, the major axis of the capsule deformation is oriented at 45° with respect to the undisturbed streamlines. In addition, the membrane with a finite viscosity reaches a steady ellipsoidal shape with its orientation angle of the major axis aligned between 45° and 0°. The orientation angle decreases with an increasing shear rate. Thus, it can be expected that in the presence of the electric and imposed flow fields the deformation and orientation behavior depend on the ratio of shear-flow forces to electric forces.

In this section, we considered the influence of the electric field on the deformation and orientation of an initially spherical capsule suspended in a simple shear flow. The solution of (26) and (27) will be sought for both the steady extensional flow induced by the electric field and the imposed linear shear flow. In this case, the nonzero components of the second-order tensors $\mathbf{E}$, $\mathbf{E}^h$, and $\Omega$ are

$$E_{11} = -\frac{1}{2}E_{22} = E_{33} = -\frac{1}{2} \quad \text{and} \quad E_{12}^h = E_{21}^h = \Omega_{12} = -\Omega_{21} = \frac{1}{2}. \quad (37)$$

Consequently, only 11, 22, 33, and 12 (and 21) components of $\mathbf{J}$ and $\mathbf{K}$ are nonzero. In addition, $J_{11} + J_{22} = -J_{33}$ and $K_{11} + K_{22} = -K_{33}$ are required. The equation of the capsule profile is obtained from (10), and can be expressed as

$$r = 1 + f = 1 + \frac{\mathbf{w}}{2}[J_{12}x_1x_2 + J_{11}(x_1^2 - x_2^2) + J_{22}(x_2^2 - x_3^2)] + O(\mathbf{w}^2) \quad (38)$$
For an illustrative purpose, let us consider first the neo-Hookean membrane. By substituting the values of $a_1$, $E$, $E^h$ and $\mathbf{\Omega}$ into (26) and (27), we obtain the following system of equations at leading order:

$$\beta \frac{\partial J_{11}}{\partial t} = -J_{11} + \beta J_{12} + K_{11} - \frac{1}{3} A_1,$$

$$\beta \frac{\partial J_{12}}{\partial t} = -\beta J_{11}/2 + \beta J_{22}/2 - J_{12} + K_{12} + \frac{5}{2} (1 + \beta),$$

$$\beta \frac{\partial J_{22}}{\partial t} = -J_{22} - \beta J_{12} + K_{22} + \frac{2}{3} A_1,$$

$$\beta \frac{\partial K_{11}}{\partial t} = -\frac{1}{3} K_{11} + \beta K_{12} - \frac{1}{6} A_2,$$

$$\beta \frac{\partial K_{12}}{\partial t} = -\beta K_{11}/2 + \beta K_{22}/2 - \frac{1}{3} K_{12} + \frac{5}{4} (1 + \beta),$$

$$\beta \frac{\partial K_{22}}{\partial t} = -\frac{1}{3} K_{22} - \beta K_{12} + \frac{1}{3} A_2,$$

$$\beta \frac{\partial K_{33}}{\partial t} = -\frac{1}{3} K_{33} - \frac{1}{6} A_2.$$

The steady-state solution is given as

$$K_{11} = \frac{45\beta (1 + \beta)}{4(1 + 9\beta^2)} + \frac{9\beta^2 - 2}{4(1 + 9\beta^2)} A_2,$$

$$K_{12} = \frac{15(1 + \beta)}{4(1 + 9\beta^2)} + \frac{9\beta}{4(1 + 9\beta^2)} A_2,$$

$$K_{22} = \frac{45\beta (1 + \beta)}{4(1 + 9\beta^2)} + \frac{9\beta^2 + 4}{4(1 + 9\beta^2)} A_2,$$

$$J_{11} = \frac{30\beta (1 + \beta)(7 + 9\beta^2)}{12(1 + \beta^2)(1 + 9\beta^2)} + \frac{9\beta^3 + 19\beta^2 - 2}{4(1 + \beta^2)(1 + 9\beta^2)} A_2$$

$$+ \frac{\beta^2 - 2}{6(1 + \beta^2)} A_1,$$

$$J_{12} = \frac{5(1 + \beta)(5 + 9\beta^2)}{4(1 + \beta^2)(1 + 9\beta^2)} + \frac{3\beta}{(1 + \beta^2)(1 + 9\beta^2)} A_2$$

$$+ \frac{\beta}{2(1 + \beta^2)} A_1,$$

$$J_{22} = -\frac{30\beta (1 + \beta)(7 + 9\beta^2)}{12(1 + \beta^2)(1 + 9\beta^2)} + \frac{9\beta^3 + \beta^2 + 4}{4(1 + \beta^2)(1 + 9\beta^2)} A_2$$

$$+ \frac{\beta^2 + 4}{6(1 + \beta^2)} A_1.$$

Again, when $\xi$ is asymptotically large, the solutions obtained here reduce exactly to those of Barthes-Biesel and Ogier.\(^7\)

For a RBC-type membrane, $K$ is linearly proportional to $J$ at the leading order, as shown in the previous section. The relationship (34) leads to a single differential equation,

$$\beta \dot{J} = \frac{12}{5} \xi (1 + \beta) E^h + \frac{1}{2} A_2 E - \frac{1}{2} J.$$

At equilibrium, the shape and orientation of the RBC-type capsule can be determined from

$$J_{11} = \frac{45\beta (1 + \beta)}{6(1 + 4\beta^2)} + \frac{3(-1 + 2\beta^2)}{6(1 + 4\beta^2)} A_2,$$

$$J_{12} = \frac{15(1 + \beta)}{4(1 + 4\beta^2)} + \frac{6\beta}{4(1 + 4\beta^2)} A_2,$$

$$J_{22} = \frac{45\beta (1 + \beta)}{6(1 + 4\beta^2)} + \frac{6(1 + \beta^2)}{6(1 + 4\beta^2)} A_2.$$

Finally, let us consider the purely viscous membrane for which the surface viscosity produces the dominant effect over the negligible elasticity. From (26) and (27), the evolution equation becomes

$$\dot{J} = s_1 E^h + s_2 E + \psi \beta^{-1},$$

with $s_1 = 5\xi$ and $s_2 = A_1/1 + \beta$ for a neo-Hookean membrane and with $s_1 = 15\xi/4$ and $s_2 = 3A_2/4(1 + \beta)$ for the RBC-type membrane. The solution is

$$J_{11} = \frac{1}{2} s_1 (1 - \cos t) + \frac{1}{2} s_2 (t - 3 \sin t),$$

$$J_{12} = \frac{1}{2} s_1 \sin t + \frac{1}{2} s_2 (1 - \cos t),$$

$$J_{22} = -\frac{1}{2} s_1 (1 - \cos t) + \frac{1}{2} s_2 (t + 3 \sin t).$$

In the presence of the imposed shear flow, the resulting capsule profile loses its axisymmetry, and therefore, the deformed capsule shape should be expressed by more than one quantity. However, in order to look into the electrohydrodynamic effect on the capsule deformation, it is sufficient to examine the meridian of the capsule profile in the 12-plane since the degree of capsule deformation is expected to be seen effectively on the 12-plane. Here, we define the degree of capsule deformation as

$$D_{12} = \frac{z_{\max} - z_{\min}}{z_{\max} + z_{\min}},$$

in which $z_{\max}$ and $z_{\min}$ are the half-lengths of the major and minor axes of the meridian profile in the 12-plane, as shown in Fig. 6. In this description, the orientation angle $\phi$ is defined as the oblique angle of the major axis relative to the flow direction. Unlike the problem considered in the previous section, the major axis in the presence of the flow is neither parallel nor perpendicular, in general, to the electric-field direction.

Now, let us then consider the prolate-type deformation of a capsule created by the superposition of the electric field onto the shear flow. In Fig. 7, the orientation angle and degree of deformation of the purely elastic RBC-type capsule are plotted as a function of $\kappa^{-1} (= eV^2/E^2)$ when the capsule is subjected to the electric field and a simple shear flow simultaneously. In this plot, the Capillary number for the imposed shear flow is fixed at $Ca = 0.05$. As discussed previously, the major axis of capsule deformation is oriented $45^\circ$ with regard to the streamlines when the electric field is negligible. It can be seen that as the electric field strength increases, the orientation angle of the major axis approaches
90° and tends to align to the electric field direction. In addition, due to the additional deforming force from the electric field, the degree of capsule deformation increases with $k^{-1}$. It is not surprising that the small deformation assumption loses its validity when $k^{-1}$ is large. In Fig. 8, the orientation angle and degree of deformation are plotted as a function of $k^{-1}$ when the electric-field-induced flow is biaxially straining. In this case, the drop would deform into an oblate spheroid in the absence of the shear flow. When the electric field is very weak, the deformation behavior is mainly governed by the shear flow. As the electric field becomes strong, the major axis of capsule deformation is gradually declined from 45° to 0°, which can be seen in Fig. 8. As expected, the degree of the capsule deformation is also increased by the electric field. However, the drop shape is transformed from a prolate type to an oblate one when the electric field contribution dominates the shear flow effect.

Now let us then turn to the shear flow effect for a given electric field strength. In Fig. 9, the orientation angle and the degree of deformation $D_{12}$ of the capsule experiencing a prolate-type deformation in an electric field are illustrated as a function of $\xi$ (or equivalently the strength of the shear flow). In this plot, the electric field strength is given by $k^{-1}=0.2$. Thus, Fig. 9 is a complement to Fig. 7. Indeed, the major axis is oriented parallel to the electric field and
perpendicular to the streamlines when the electric field is dominant. By increasing the strength of the shear flow (or equivalently increasing $\xi$), the orientation angle approaches $45^\circ$ and the degree of deformation increases. On the other hand, when the capsule experiences an oblate-type deformation in the absence of an imposed shear flow, the major axis is perpendicular to the electric field, while it is parallel to the streamlines, as can be seen in Fig. 10. When the shear flow is weak, the resulting capsule profile is an oblate spheroid. However, because $\kappa^{-1}$ is small, its deviation from sphericity is not considerable. It is obvious that the orientation angle approaches again $45^\circ$ and the degree of deformation increases as the shear flow becomes predominant.

In Fig. 11, the effect of the membrane viscosity on the capsule orientation is included for the RBC-type membrane when the electric field effect is dominant over the shear flow ($\xi=0.1$). In fact, the dynamic responses of a capsule are governed by two dimensionless physical variables $\kappa$ and $\beta$ (or $\eta$). Thus, we must specify a priori some of the physical properties for a specific capsule in order to plot the degree of deformation as a function of $\kappa^{-1}$. Here, we choose the capsules with the physical properties similar to those of RBC, i.e., the capsules of 5 $\mu$m in radius suspended in a fluid of the viscosity about 0.05 Pa s. In this case, $\eta=2$. Like a typi-

**FIG. 9.** The effect of a simple shear flow on the orientation angle $\phi$ and the degree of deformation $D_{12}$ of the capsule experiencing a prolate-type deformation in an electric field. The capsule is enclosed by a purely elastic RBC-type membrane ($\beta=0$) and the parameters denoting electrical properties are fixed at $R=0.1$ and $S=1$. The electric field strength is given by $\kappa^{-1}=0.2$.

**FIG. 10.** The effect of a simple shear flow on the orientation angle $\phi$ and the degree of deformation $D_{12}$ of the capsule experiencing an oblate-type deformation in an electric field. The capsule is enclosed by a purely elastic RBC-type membrane ($\beta=0$) and the parameters denoting electrical properties are fixed at $R=10$ and $S=1$. The electric field strength is given by $\kappa^{-1}=0.02$.

**FIG. 11.** The effect of a uniform electric field on the orientation angle $\varphi$ of the major axis of the deformed capsule as a function of $\beta$. The capsule is enclosed by a viscoelastic, RBC-type membrane for $S=1$ and $\xi=0.1$. 

Downloaded 12 Nov 2009 to 143.248.125.44. Redistribution subject to AIP license or copyright; see http://pof.aip.org/pof/copyright.jsp
The effect of uniform electric field on the orientation and deformation of the capsule with a viscoelastic membrane is considered analytically. In order to pursue the analytic procedure, the deformation of the capsule is assumed to be small. The electric-field-induced flow is considered on the basis of the leaky dielectric theory. A regular perturbation solution is obtained when the particle is initially spherical and is undergoing small deformations. First, purely electric-field-induced deformation is considered in the absence of an imposed external flow. The deformation type is determined by the electrical properties of the fluids inside and outside the capsule as in the case of an ordinary interfacial-tension drop. The present analysis shows that the electric-field-induced deformation is not influenced by the surface viscosity of the membrane as well as the viscosity ratio. Then, a capsule immersed in a simple shear flow is considered. In this case, the surface viscosity affects the degree of deformation while the viscosity ratio still does not have any influence. Further, the contribution from the ambient vorticity cannot be ignored, even at leading order. In the case of an elastic or a viscoelastic capsule, the effect of electric field on the orientation angle of the capsule is considerable. For a purely viscous capsule, the orientation angle oscillates whether the electric field is applied or not. However, due to the irrotational contribution from the electrohydrodynamic flow, the capsule deformation continuously increases with time. In spite of the inherent limitation of analytical procedure, the asymptotic theory agrees well with the experimental observation of the orientation of an interfacial-tension drop which is under the simultaneous actions of the electric and flow fields.

**ACKNOWLEDGMENTS**

This work was supported partly by a grant from the Ministry of Science and Technology and by a contracted research project with Hyundai Heavy Industries Co., Ltd. The authors appreciate their support. One of the referees pointed to one very important issue regarding the boundary conditions, which was essential to correct the asymptotic analysis.

20. J.-W. Ha and S.-M. Yang, “Effect of nonionic surfactant on the deforma-
tion and breakup of a drop in an electric field,” J. Colloid Interface Sci. 206, 195 (1998).