Supporting Universal Quantification in a Two-Dimensional Database Query Language

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Abstract: We propose a technique for specifying universal quantification and existential quantification (combined with negation) in a two-dimensional (graphical) database query language. Unlike other approaches that provide set operators to simulate universal quantification, this technique allows a direct representation of universal quantification. We present syntactic constructs for specifying universal and existential quantifications, two-dimensional translation of universal quantification to existential quantification (with negation), and translation of existentially quantified two-dimensional queries to relational queries. The resulting relational queries can be processed directly by many existing database systems. Traditionally, universal quantification has been considered a difficult concept for typical database programmers. We claim that this technique renders universal quantification easy to understand. To substantiate this claim, we provide a simple, easy-to-follow guideline for constructing universally quantified queries. We believe that the direct representation of universal quantification in a two-dimensional language is new and that our technique contributes significantly to the understanding of universal quantification in the context of database query languages.

1.0 Introduction

Universal quantification is an important element in relational calculus [Codd71]. Yet, it has not been fully integrated in many practical database query languages. There are two possible reasons: 1) in a linear-synax language, complex syntax is needed to support universal quantification, and 2) universal quantification can be replaced with existential quantification and negation, which many languages provide. Some approaches support universal quantification by using set operators [Zok75] [Ozas87] However, in these approaches, the user has to transform a universally quantified query to multiple subqueries connected by set operators. Sometimes, the transformation is a non-trivial task for average database programmers. SQL [IBM86] supports universal quantification that can be specified in the form \texttt{SELECT} \texttt{ALL} \texttt{subquery}. However, only very limited cases of universal quantification can be represented in this form.

In this paper we present a simple, elegant technique for specifying universal quantification. Our technique employs a two-dimensional representation of queries. Unlike other set-oriented approaches, the technique allows a direct representation of universal quantification. We first present syntactic constructs for specifying universal quantification and existential quantification (with negation). We then present an algorithm for transforming automatically a universally quantified query to an existentially quantified query. Next, we present an algorithm to transform an existentially quantified query to a relational calculus query. This sequence of transformations proves that the universally quantified query specified in our two-dimensional language can be easily implemented by using any of many existing relational database systems provided that it supports negation and existential quantification. (Many database systems support existential quantification implicitly or explicitly. See Section 4 for more discussion on this aspect.)

Many two-dimensional query languages have been proposed in the literature [Zok75] [McD75] [Won82] [Zha83] [Elm85] [Kim88]. We examine these languages by classifying their features into three categories: the data model, aggregation, and quantification. We pay special attention to aggregation and quantification because these features require a scoping operator to define parts of the query (i.e., subqueries) to which they apply. In a linear syntax, the scoping operator is a parenthesis or a keyword. In a two-dimensional syntax, it will be a box or an enclosure. As we discuss in subsequent sections, we use boxes to represent quantifications. Aggregation requires a scoping operator when it appears in certain conditions as exemplified in [McD75].

A pioneering work in two-dimensional representation of database queries is Query-by-Example (QBE) [Zok75]. A language based on the relational model, it supports aggregation and existential quantification (with negation). It also supports universal quantification by using a set notation. Due to the lack of the scoping operator (i.e., subqueries cannot be defined), however, ambiguity can arise if aggregation appears in a condition or if quantification involves more than one relation. CUPID [McD75] is also based on the relational model and has features similar to QBE's. However, it does not provide the mechanisms to specify quantification although it does provide a scoping operator to specify subqueries involving aggregation. GUIDE [Won82] is based on the entity-relationship model, but does not support aggregation or quantification. Elmasri and Larson [Elm85] also proposed a language based on the entity-relationship model. It provides set operators and aggregation operators, but does not provide explicit scoping operators. However, it is possible to resolve ambiguity in scoping by rephrasing the queries in English and asking the user to verify them. PICASSO [Kim88] uses the universal relation model [Mal83] as its basis and supports set operators as well as a scoping operator to be used for each maximal object. The scoping operator can be used for aggregation, but not for quantification. Quantification can be handled through set operators although this aspect was not discussed explicitly in the paper. GQL/ER [Zha83] combines the features of the entity-relationship model and the universal relation model. This language does not support aggregation or quantification. Finally, Ozsoyoglu [Ozas87] proposed a linear syntax language called RC/3$. RC/3$ is a variation of relational calculus that replaces universal quantification with operations on sets.

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1. We describe our technique using the entity-relationship model because of its elegance in representing the relationship. Nonetheless, the technique is equally applicable to the relational model where relationships are replaced with join conditions.
Our query language supports aggregation, universal quantification, and existential quantification (with negation). In this paper we concentrate on the facilities for quantifications and do not discuss aggregation in depth. Here, we identify two distinct contributions of this paper. First, we claim that our quantification scheme is easy to use. Traditionally, universal quantification has been considered a difficult concept for typical database programmers. Substantiating this claim, we present a simple and easy guideline for constructing universally quantified queries. This guideline works for most of the commonly encountered queries. Second, we believe that the direct representation (without using set operations) of universal quantification in a two-dimensional language is new and it contributes to the understanding of universal quantification in database query languages. The class of universally quantified queries that can be expressed in our language is formally defined in Section 5. We believe that it includes most of the queries commonly encountered in practical situations.

The organization of the paper is as follows. Section 2 briefly introduces our two-dimensional query language. Section 3 presents the syntactic constructs for composing queries with universal quantification. Similarly, Section 4 presents the constructs for existential quantification with negation. Section 5 formally defines the class of universally quantified queries that we handle and presents the algorithm for transforming a universally quantified query to an existentially quantified query with negation. Section 6 presents the algorithm for transforming an existentially quantified query to a relation calculus query. We present a simple guideline for composing universally quantified queries in Section 7 and discuss a more complex case in Section 8. Finally, we conclude the paper in Section 9.

2.0 A Two-Dimensional Query Language

In this section we briefly introduce our two-dimensional database query language. We present only those features that are relevant for the discussions in this paper. A full description of the language will be presented in a future paper.

A query is a specification of conditions according to which entities are selected from among those contained in the database. We define a schema diagram as a graph that represents the structure of a database. We use the entity-relationship (ER) model [Che76] for its basis. A schema diagram consists of three constructs: entity sets, one-to-many (including one-to-one) relationship sets, and many-to-many (including nonbinary) relationship sets. An entity set appears as a rectangular node with the name of the entity set in it. A one-to-many relationship set appears as an arc with the name of the relationship in the middle. An end of the arc adorned with the symbol "*" represents a cardinality of 'many', while an undecorated end represents a cardinality of 'one'. A many-to-many relationship set or a nonbinary relationship set appears as a rhombus node with the name of the relationship in it. We draw undecorated arcs between the rhombus and the entity sets participating in the relationship.

A query graph is a subgraph of the schema diagram with possibly certain nodes and arcs replicated. In addition, each node of the query graph can have logical conditions and projection information associated with it. There is also a global condition box in which complex conditions can be specified. We classify logical conditions into three categories: a selection condition that applies to a single node, a join condition that applies to a set of nodes, and an aggregation condition that involves an aggregation operation. These conditions are specified in an area called a query box. For each node, one or more query boxes can be created by clicking the mouse with the cursor positioned on the node. For the purpose of this paper, however, we simply write the condition next to the node without using a query box. Thus, we write a selection condition next to the node representing the entity set to which the condition applies. We do not discuss aggregation conditions because they are beyond the scope of this paper. We select a projection attribute by clicking on the attribute name in the query box. Selected projection attributes are shown in reverse video. In this paper, for simplicity and without loss of generality, we assume all the attributes of the entity set (rather than a subset of the attributes) are projected. We indicate projection by writing the symbol "proj" next to the entity set.

In Figure 1 we illustrate the use of these constructs by using a simple query. The query states "List the employees whose salaries are more than one tenth of the budget of their department and who participate in a project that has more than ten members."

![Figure 1. An Example Two-Dimensional Query.](image)

The query contains three entity sets, Dept, Emp, and Project: a one-to-many relationship set, employ; and a many-to-many relationship set, Participate. A selection condition is specified for the entity set Project, and a join condition is specified for the entity sets Emp and Dept. The result of the query is projected from the entity set Emp.

3.0 Universally Quantified Queries

In this section we present how universally quantified queries are expressed in our two-dimensional query language. Consider the following query: "List the departments that sell all the items supplied by the supplier Parker." In relational calculus, the query is represented as follows:

\[
\{D \mid \exists x(D, x) \wedge (D \subseteq X) \wedge \forall y(S, y) \wedge \forall z(S, z) \wedge (D \cap X = Y) \wedge (x = Y) \}
\]

For convenience, we assume in this section that an entity set or a relationship set is mapped to a relation. In Section 6, we relax this restriction by mapping a one-to-many relationship set to a foreign key without representing it as a separate relation. The query is represented in Figure 2, where a universal quantification box (U-box) drawn with bold lines encloses universally quantified variables, I(for Item), S(for Supplier), and D(for Supplier).

![Figure 2. A Query with Universal Quantification.](image)
Note that the query in Figure 2 is different from the query in Figure 3, which says "List the departments such that all the items they sell are supplied by the supplier Parker." In relational calculus this query is represented as

\[(T_1 \equiv \exists D(DT \land DT[2] = 1) \land \\
\exists_{\text{Item}(i) \land \text{Sell}(i, D) \land \text{Sell}(i, S)}(S, i) = 1) \land \\
\exists_{\text{Supplier}(i) \land \text{Supply}(i, S) \land \text{Sell}(i, S)}(S, i) = 1) \land \\
\text{Sell}(i, S) = 1 \land S[2] = \text{Parker})\]

In this query, the phrase "they sell" modifies the noun (items) that is universally quantified, thus composing a noun phrase. In the relational calculus representation, the variables corresponding to this noun phrase are S(for Sell) and I(for Item). These variables are universally quantified because they represent the universally quantified noun and the conditions associated with it. Thus, we enclose Sell and Item in a U-box.

![Figure 3. Another Query with Universal Quantification.](image)

In Figure 4 and Figure 5, we present two additional examples of universally quantified queries. The schema diagrams in these examples contain one-to-many and ternary relationship sets whereas those in Figure 2 and Figure 3 contain many-to-many relationship sets. We use these schema diagrams throughout the paper for illustrative purposes.

The query in Figure 4 states "List the divisions where all the departments they own have at least one employee whose salary is greater than 50000 dollars." In relational calculus, it is represented as:

\[(T \equiv \exists D(DV \land DV = T) \land \\
\exists_{\text{Emp}(E) \land \text{Employ}(E, D)}(E, D) \land \\
\exists_{\text{Salary}(S) \land \text{Salary}(S, E)}(S, E) \land \\
\text{Salary}(S) > 50000)\]

Note that the universally quantified noun phrase is "the departments they own." Thus, we enclose the entity set Dept and the relationship own in the U-box.

![Figure 4. A Universally Quantified Query with One-to-Many Relationship Sets.](image)

The query in Figure 5 states "List the suppliers that supply all the parts of type A to companies located in New York." The universally quantified noun phrase is "the parts of type A." Thus, the U-box encloses the entity set Item with the condition Type=A.

![Figure 5. A Universally Quantified Query with a Ternary Relationship Set.](image)

### 4.0 Existential Quantification and Negation

We discuss in this section how existential quantification is specified in our two-dimensional query language. Existential quantification is implicitly supported by many relational query languages. For example, consider the SQL query "SELECT dept FROM dept, emp WHERE dept.dno = emp.dno AND emp.salary > 50000." This query can be represented in relational calculus as follows:

\[(T \equiv \exists E(DT \land DT[2] = 1) \land \\
\exists_{\text{Emp}(E) \land \text{Emp}(E, D)}(E, D) \land \\
\text{Emp}(E) \land E[2] > 50000)\]

Note that the existential quantification on DT and E is implicit in the SQL query. In these query languages, however, existential quantification is made explicit when negation is involved. For example, consider a SQL query "SELECT * FROM dept WHERE NOT EXISTS (SELECT * FROM emp WHERE X.dno = emp.dno AND emp.salary > 50000)." This query returns the dept tuples only when there is no employee in the dept who earns more than 50000 dollars. The query is represented in relational calculus as follows:

\[(T \equiv \exists_{\text{Emp}(E) \land \text{Emp}(E, D)}(E, D) \land \\
\exists_{\text{Salary}(S) \land \text{Salary}(S, E)}(S, E) \land \\
\text{Salary}(S) > 50000)\]

With an existential quantifier, associated is a scope within which the quantification is effective. For example, in the query

\[(T \equiv \exists_{V[2]}(V[1] = 1) \land \\
\exists_{V[2], V[3]}(V[2] \land V[3]) \land \\
\exists_{V[1]}(V[2] \land V[3])\)

the scopes of existential quantification are enclosed by the brackets. In a two-dimensional language, we represent a scope by a two-dimensional bracket, i.e., a box. In our language, we allow use of explicit existential quantification only when it is used in conjunction with negation. Thus, a box for negated existential quantification (NE-box) represents NOT EXISTS (a subquery) in the SQL syntax. The use of this NE-box (drawn with broken lines) is illustrated in Example 1 and Example 2.

#### Example 1:

Consider the query, "List the departments where none of the employees in the department have a salary of more than 50000 dollars." In our two-dimensional query language, the query is expressed as in Figure 6.
5.0 Translation of a Universally Quantified Two-Dimensional Query to an Existentially Quantified Two-Dimensional Query

In this section we describe how we translate automatically a universally quantified query that the user composes into an existentially quantified query. We present a translation algorithm and show its correctness.

Universally quantified queries in our language are in the following general form.

\[
\{T \mid \exists Y(P(V_1, V_2) \land \forall Y(Q(V_1, V_2) \rightarrow \exists Z(R(V_1, V_2, Z)))\}
\]

(5)

where \(P, Q, \) and \(R\) are formulas, \(V_1, V_2,\) and \(V_3\) are sets of tuple variables, and \(T\) is a set of free variables. Free variables represent the tuples that appear in the result of the query, i.e., the tuples that are projected. For example, the query in Figure 2 on page 4 was expressed as in Eq.(1).

We define a scope to be a set of entity sets, relationship sets, and conditions. In Eq.(5) a scope corresponds to a set of tuple variables and formulas. We define three different scopes. For convenience, we define an entity set, a relationship set, or a logical condition as an element.

1. Scope 1: This scope includes the entity sets that are projected plus any other elements that are not included in Scopes 2 and 3. In Eq.(5), Scope 1 includes the tuple variables in \(V_1\) and \(T\) plus the formula \(P(V_1, V_2).\)

2. Scope 2: This scope includes the elements enclosed by the U-box (i.e., universally quantified elements). In Eq.(5), Scope 2 includes the tuple variables in \(V_2\) plus the formula \(Q(V_1, V_2).\)

3. Scope 3: Consider a reduced graph where the projected entity sets are eliminated. Scope 3 includes the elements that are directly or indirectly connected to those in Scope 2 in the reduced graph. In Eq.(5), Scope 3 includes the tuple variables in \(V_3\) plus the formula \(R(V_1, V_2, V_3).\)

Example 3 illustrates how we identify different scopes.

Example 3:

In Figure 4 the entity set Div belongs to Scope 1, the entity set Dept and the relationship set own belong to Scope 2, and the entity set Emp, the relationship set employ, and the condition Salary > 50000 belong to Scope 3. Suppose the query is slightly modified as in Figure 8. Then, the relationship set rel and the entity set Ent also belong to Scope 1. Note that they do not belong to Scope 3.

![Figure 7. A Query with Nested Existential Quantification with Negation.](image)

![Figure 8. An Example Query for Identifying Scopes.](image)
We now present the algorithm for translating a universally quantified query to an existentially quantified query.

Algorithm 1 (U-to-E Translation):
1. Put an NE-box around all the elements in Scopes 2 and 3.
2. Put an NE-box around all the elements in Scope 3. Note that this box is completely enclosed by the NE-box in Step 1. If no element exists in Scope 3, create an element with the value of "true" and put an NE-box around it.

Correctness of the Translation Algorithm

Algorithm 1 essentially reflects the following equality:

$$\forall \alpha \implies \exists \beta \equiv \forall \neg \alpha \lor \exists \beta \equiv \neg \exists (\alpha \land \neg \beta).$$

Using this equality, Eq. (5) can be transformed as follows:

$$\{ \forall \exists \beta (P(V', T) \land \neg \exists \beta (Q(V', F2) \land \exists \beta (R(V', V2, F3)))) \}$$

Eq. (7) indicates that an NE-box is applied to all the elements in Scopes 2 and 3. In addition, another NE-box is applied to all the elements in Scope 2. This proves the correctness of the translation algorithm.

We illustrate this translation in Examples 4 and 5.

Example 4:

The query in Figure 4 is transformed from Eq. (3) as follows:

$$\{ \forall \exists \beta (Dm(DV) \land (DV = T)) \land \neg \exists \beta \rho (D(DT) \land \mu(DM) \land (O1 = DT(1) \land O2 = DT(2) \land \neg \exists \mu(DM) \land \mu(DM) \land (H1 = DT(1) \land H2 = DT(2) \land (E2 > 50000))) $$

Eq. (8) corresponds to the equivalent existential query in Figure 7.

Example 5:

The query in Figure 5 is translated into an existentially quantified query in Figure 9. The query states "List the suppliers for which there are no parts of type A that they do not supply to companies located in New York."

![Diagram](image)

Figure 9. An Existentially Quantified Query with a Ternary Relationship Set.

6.0 Translation of an Existentially Quantified Two-Dimensional Query to a Relational Calculus Query

In Section 5, we discussed how a universally quantified query can be translated to an existentially quantified query with negation. In this section, we present an algorithm for translating an existentially quantified query to a tuple relational calculus query. Using this transformation, a universally quantified query can be easily implemented by using existing relational database systems that support only existential quantification with negation.

To translate the query, we first need to translate the schema according to the underlying data model. The translation of an entity-relationship model schema to a relational model schema is well known [UB82]. Here, we adopt a translation technique using system-generated identifiers, i.e., surrogates. We briefly review basic techniques for schema translation and then present query translation.

6.1 Schema Translation

For schema translation, we introduce two types of relations: entity relations and relationship relations. First, for each entity set, we create a relation scheme (entity relation) that consists of all the attributes of the entity set plus a surrogate attribute and foreign key attributes. The surrogate uniquely determines the tuple. A foreign key attribute is added for each one-to-many relationship set in which this entity set is on the many-side of the relationship. The foreign key attribute is the surrogate attribute of the relation on the one-side of the relationship. We treat a one-to-one relationship set like a one-to-many relationship set, adding a foreign key attribute to one of the entity sets. We treat the entity set so chosen as if it were the one on the many-side of the one-to-many relationship. Second, for each many-to-many or nonbinary relationship set, we create a relation scheme (relationship relation) that consists of the surrogate attributes of the entity sets participating in the relationship.

6.2 Query Translation

We now present the query translation algorithm. We describe it in two steps: We first consider queries without NE-boxes. Then, we consider queries containing NE-boxes.

6.2.1 Queries without NE-boxes

Algorithm 2 (Simple-Translation):
Input: A two-dimensional query without NE-boxes
Output: A tuple relational calculus query

Constructing a relational query in this case is straightforward; thus, we only sketch the algorithm. First, we construct an atom of the form \(R(V)\) for each entity set, many-to-many relationship set, or nonbinary relationship set, where \(R\) is the name of the relation corresponding to the entity set, many-to-many relationship set, or nonbinary relationship set, and \(V\) is the tuple variable. We say that the formula \(R(V)\) defines the tuple variable \(V\). Second, we construct a formula of the form \(V1[A1] \equiv V2[A2]\) for each one-to-many relationship set, where \(V1, V2\) are tuple variables for the relations on either side of the relationship, \(A1\) is the positional index for the surrogate attribute of the relation on the one-side of the relationship, and \(A2\) is the positional index for the foreign key attribute of the relation on the many-side of the relationship. Similarly, we construct two equality formulas for each many-to-many relationship set equating the tuple variable for each of the two entity relations and the tuple variable for the relationship relation via the surrogate and foreign key attributes. For a nonbinary relationship set involving \(n\) entity sets, we construct \(n\) equality formulas. Third, we construct an appropriate formula for each condition specified. Fourth, all these formulas are logically ANDed, and the result is quantified by \(\exists_{\alpha}\), where \(\alpha\) is a tuple variable defined in the formulas. Last, we equate each tuple variable to be projected with a free variable \(V\). Example 6 illustrates this algorithm.
Example 6:

Consider the query in Figure 10: "List the suppliers who supply a part of type A to a company located in New York."

![Figure 10. A Query with a Ternary Relationship Set.](image)

The corresponding relational query is as follows:


Here, S, C, P, and Su are tuple variables, S[1], C[1], P[1], and C[2] represent surrogate attributes of relations Supplier, Company, and Part, and S[2], S[3], and C[2] represent foreign key attributes of the relation Supply. The first three equality formulas come from the ternary relation set Supply and the last two come from the conditions for the entity sets Company and Part.

6.2.2 Queries with NE-boxes

We use the notation \( Q_{\text{outer}} \) to represent the part of the query \( Q \) that is outside the outermost NE-boxes within \( Q \). We call the part of the query within an NE-box as \( Q_{\text{inner}} \). If a relationship name appears within the NE-box, the relationship set is part of \( Q_{\text{outer}} \) even if it may be connected to an entity set outside the NE-box. The parameter \( n \) is the number of outermost NE-boxes in \( Q \).

Algorithm 3 (Translation):

Input: A two-dimensional query \( Q \) with zero or more NE-boxes
Output: A tuple relational calculus query

Begin

\[ G = \text{Simple-Translation}(Q_{\text{outer}}) \]

For each outermost NE-box of \( Q \)

\[ F_i = -3_{\text{outer}}(\text{tuple variables of } F_i \text{ and } Q_{\text{inner}}) \text{ Translation } (Q_{\text{inner}}) \]

End

In a formula \( F_i \), we do not generate existential quantification if \( (Q_{\text{inner}}) \) has no tuple variables defined in it. In this case, the subquery simply becomes a condition. Note that Algorithm 3 is called recursively for the subqueries within NE-boxes. In translating a subquery, all the tuple variables defined outside its scope can be referenced. For example, in Example 7, the tuple variable \( DT \) is referenced within the innermost subquery.

Example 7:

Consider the query in Figure 7. The translated tuple relational calculus query is as follows:


7.0 A Guideline for Composing a Universally Quantified Query

Writing a universally quantified query is often not intuitively obvious. Thus, we present a simple guideline for composing a universally quantified query. We present this guideline for the following reasons:

1. The concept of universal quantification is more complex than most other concepts in a query language. We believe that this complexity is inherent and is not specific to a query language.
2. Even when a query does not involve universal quantification, the flexibility of a natural language (e.g., words like "all") can give the impression that the query does involve universal quantification, as we explain in the guideline below.

Guideline (U-query):

1. Rewrite the query in English by eliminating any occurrences of "all" (or "each," "every," "at least") where the elimination does not alter the meaning of the query. For example, consider the query, "List all the divisions where all departments they own have an employee whose salary is greater than 50000 dollars." In this query, the first "all" can be removed without altering the meaning of the query, but the second "all" cannot. Thus, the reduced query is "List the divisions where all the departments they own have an employee whose salary is greater than 50000 dollars."

2. Identify the noun phrase that is quantified by the word "all." A noun phrase includes the noun quantified by the word "all" and the phrase (if any) that modifies the quantified noun. In the example above, the noun phrase is "the departments they own."

3. Compose a query as if the word "all" were replaced by an indefinite article. In the example above, we construct the query, "List the divisions where a department they own has an employee whose salary is greater than 50000 dollars." Construct the corresponding query graph.

4. Put a U-box around the entity sets, relationship sets, and logical conditions that correspond to the noun phrase identified in Step 2. Thus, the example query is represented as in Figure 4.

8.0 Queries with Implicit Projection

In Section 7 we discussed a basic guideline for composing a universally quantified query. In this section we discuss a case that needs special attention.

A query may have different meanings depending on whether certain elements belong to Scope 1 or Scope 3. For example, consider the query in Figure 11.
Figure 11. A Query Becomes Ambiguous if Implicit Projection is Allowed.

The query states "List the divisions such that for each blue item there is a department in the division which sells the item." Note that the query (Query1) is different from the following query (Query2): "List the divisions owning a department that sells all the blue items." Query1 qualifies a division if the departments it owns collectively cover all the blue items, while Query2 requires that a single department cover all the blue items.

The scoping rules in Section 5 interpret the query as Query1 since the entity set Dept and the relationship set own are contained in Scope 3. To interpret the query as Query2, we have to assume that there is an implicit projection on the entity set Dept, which will put the entity set Dept and the relationship set own in Scope 1.

Implicit projection makes the query ambiguous. To disambiguate it, we have two alternatives: 1) to provide a syntactic construct to distinguish Scope 1 from Scope 3 explicitly or 2) to disallow implicit projection by requiring that the entity set Dept be projected as well. We chose the latter option for the simplicity of the scoping rules and for ease of use. We believe that this requirement is reasonable since in Query2 the user would quite likely be interested in having in the query result the specific department that covers all the blue items.

9.0 Summary

We have presented a technique for specifying universal quantification and existential quantification (with negation) in a two-dimensional database query language. Our technique allows a direct representation of universal quantification in a two-dimensional manner without using set operators. We have also presented a two-dimensional algorithm to transform a universally quantified query to a query with existential quantification and negation, and showed its correctness. Finally, we have presented an algorithm to transform an existentially quantified query to a relational calculus query. This transformation allows the universally quantified queries to be easily processed by many existing database management systems that support existential quantification with negation.

Universal quantification has been considered a difficult concept in database query languages. We claim that our technique renders the concept easy to understand. Substantiating this claim, we have presented a simple, easy-to-follow guideline for constructing queries with universal quantification.

We believe that the technique of directly representing universal quantification without using sets in a two-dimensional query language is new and that its ease of use will contribute to bring the concept of universal quantification more into the world of database query languages.

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References


Appendix I Safety of Queries

In this appendix we briefly discuss the safety issue. The safety issue was discussed extensively in [Dem82] [Van87] [Kri86] [She86]. We omit detailed proofs and discussions on safety since they are beyond the scope of the paper.

A query (or a formula) is safe if it has a finite result. A class of formulas called evaluable formulas defined by Demolombe [Dem82] and refined by Van Gelder and Topor [Van87] is by far the largest known decidable subset of safe formulas. A class of allowed formulas is a subset of evaluable formulas whose intermediate results are finite as well. Thus, allowed queries ensure safe execution to produce the results.

It can be shown that any relational calculus formula in the form of Eq. (5) is not evaluable if a free variable appears in Scope 2 or Scope 3, i.e., in formulas \( Q \) and \( R \). For example, the following queries are not evaluable (and in this case unsafe):

\[
\begin{align*}
|T | \exists y_1 \exists y_2 (Q(y_1,y_2,y_3,y_4) = \exists y_5 (R(y_5,y_4,y_3,y_2))) \\

\text{[9]} \\

|T | \exists y_1 \exists y_2 y_3 (Q(y_1,y_2,y_3) = \exists y_4 (R(y_5,y_4,y_3,y_2))) \\

\end{align*}
\]

In Eq. (9) the universally quantified subformula is satisfied regardless of \( T \) values if the second disjunct is satisfied. It is also satisfied for all (and possibly an infinite number of \( T \) values that do not satisfy the formula \( Q(y_1,y_2,y_3,y_4) \). (According to the formalism in [Van87], \( gen(T,=Q(y_1,y_2,y_3,y_4)) \) fail.) Similarly, in Eq. (10), the universally quantified subformula is satisfied regardless of \( T \) values if the first disjunct is satisfied. (According to [Van87], \( gen(T,=Q(y_1,y_2,y_3)) \) fail.) Thus, both queries can produce an infinite number of values for \( T \) and therefore are unsafe.

It can also be shown that a formula in the form of Eq. (5) is allowed (and therefore safe) if it satisfies the following conditions:

1. For every quantified subformula, each quantified variable appears in a base formula that is not contained in a nested quantification. A base formula is an atomic formula whose predicate symbol represents a database relation.
2. The subformulas \( P, Q, \) and \( R \) are conjuncts of base formulas, atomic formulas that are conditions, and universally quantified formulas of the form in Eq. (5) that satisfy this condition recursively.

We note that the definition of the allowed class of formulas presented in [Van87] does not have to be extended for equality since all the variables appear in base formulas that form conjunctions in the formulas \( P, Q, \) and \( R \).

The two-dimensional query language we present in this paper, when mapped to the relational model, satisfies the two properties with one exception. This exception is the treatment of the set of free variables \( T \). i.e., the variables in \( T \) do not appear in base formulas. Nevertheless, since they are always equated to variables in the base formulas, they do not affect the safety of the query. Hence, we can treat the queries as if they did not include these free variables. Therefore, the queries in our query language are allowed, and safe.