Estimation of Two-Dimensional DOA under a Distributed Source Model and Some Simulation Results

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SUMMARY Most research on the estimation of direction of arrival (DOA) has been performed based on the assumption that the signal sources are point sources. In some real surroundings, signal source localization can more adequately be accomplished with distributed source models. When the signal sources are distributed over an area, we cannot directly use well-known DOA estimation methods, because these methods are established based on the point source assumption. In this paper, we propose a 3-dimensional distributed signal source model, in which a source is represented by two parameters, the center angle and degree of dispersion. Then, we address the estimation of the elevation and azimuth angles of distributed sources based on the parametric distributed source modeling in the 3-dimensional space.

key words: estimation of DOA, 3-dimensional distributed source, parametric source modeling

1. Introduction

Most results on the direction of arrival (DOA) estimation problems have been obtained for the azimuth-only estimation problem based on the assumption that the signal sources are point sources [1]–[8]; i.e., if the DOA of a signal is \( \theta \), there is no other signal at \( \theta + \epsilon \), for a sufficiently small value of \( \epsilon \). It is a reasonable assumption if the signal sources are located far enough from the receivers. Under this assumption, the array output vector is a weighted sum of a finite number of steering vectors when the number of signal sources is finite, with the weighting dependent on the signal sources. As an extension of the azimuth-only estimation problem, simultaneous azimuth and elevation estimation problem was also considered in several studies under the point source model [9]–[12].

In real surroundings, the signals received at an array usually consist not only of a direct path signal, but also of multiple return signals that are coherent, phase-delayed, and amplitude-weighted replicas of the direct path signal. Among the typical examples are multiple echos in sonar, spurious returns such as clutter in radar, coherent interference due to jamming signals in satellite communication, and coherent reflections due to scatterers nearby a mobile terminal in mobile communication. In such cases, the signal source is spread around the center direction \( \theta_k \), with multiple return signals existing in the interval \( [\theta_k - \epsilon, \theta_k + \epsilon] \) on a single frequency for some nonnegligible value of \( \epsilon \). It is noteworthy that, if the signal sources are distributed, the array output vector should be expressed by integrating a steering vector over all direction of arrival with the weighting of distributed source density function [13], [14]. In addition, although the DOA estimation methods for point signal sources may be applied to the DOA estimation for distributed signal sources, it is not guaranteed that the methods would provide us with good estimates.

In this paper, we will consider simultaneous estimation of the elevation and azimuth angles of distributed signal sources: the results in this paper thus differ from those in [9]–[12] in that we consider distributed sources, not point sources. In addition, this paper is different from [13],[14] in that we consider the estimation of both azimuth and elevation angles. We consider a method to obtain the azimuth and elevation angles of distributed signal sources based on the multiple signal classification (MUSIC), and the performance of the method is investigated with some examples.

2. DOA Estimation Problems and the Signal Source Model

2.1 DOA Estimation Problems

In the azimuth-only estimation problem for the point source model with \( M \) array elements and \( L \) signal sources, the \( M \times 1 \) array snapshot (or array output) vector \( y(t) = [y_1(t), \cdots, y_M(t)]^T \) can be expressed as

\[
y(t) = \sum_{k=1}^{L} a(\theta_k)s_k(t) + n(t),
\]

where \( s_k(t) \) is the \( k^{th} \) point signal waveform, \( n(t) = [n_1(t), \cdots, n_M(t)]^T \) is the array sample noise vector, \( a(\theta_k) \) is the steering vector of the array depending on the array structure, and \( \theta_k \) is the DOA of the \( k^{th} \) source. The additive noise \( n(t) \) is assumed to be a zero-mean white complex Gaussian random vector with \( E[n(t)n^H(t)] = \sigma^2 I \) and \( E[n(t)n^T(t)] = 0 \), where \( H \) denotes the Hermitian. In addition, the noise is assumed to be uncorrelated with the signal waveforms.
Defining \( A(\theta) = [a(\theta_1), \cdots, a(\theta_L)] \) and \( \theta = [\theta_1, \theta_2, \cdots, \theta_L]^T \), (1) can be rewritten as
\[
y(t) = A(\theta)s(t) + n(t),
\]
(2)
where the zero-mean complex normal signal vector \( s(t) = [s_1(t), \cdots, s_L(t)]^T \) is stationary with covariance matrix \( E[s(t)s^H(t)] = R_s \) and \( E[s(t)s^H(t)] = 0 \). The steering vectors at different DOAs are linearly independent, i.e., the matrix \( A(\theta) \) has full rank \( L \).

Under these assumptions, the array output vector is complex Gaussian with mean zero and covariance matrix
\[
R_y = E[y(t)y^H(t)] = A(\theta)R_sA^H(\theta) + \sigma^2 I.
\]
(3)
For point sources, some aspects of the azimuth-only estimation problems have been studied in [1]–[8]. Estimation of the azimuth and elevation DOAs of point signal sources has been considered in [9]–[12] by extending (2) as
\[
y(t) = A(\theta, \phi)s(t) + n(t).
\]
(4)
In (4), \( \theta = [\theta_1, \cdots, \theta_L]^T \) is the vector of the azimuth DOAs, \( \phi = [\phi_1, \cdots, \phi_L]^T \) is the elevation DOA vector, and \( A(\theta, \phi) = [a(\theta_1, \phi_1) \ a(\theta_2, \phi_2) \ \cdots \ a(\theta_L, \phi_L)] \), with \( a(\theta_k, \phi_k), k = 1, 2, \cdots, L \), the two-dimensional steering vectors.

Recently, the azimuth-only estimation is considered under distributed source models [13], [14]. For example, consider the distributed source model in [14]. The array output vector \( y(t) \) can be expressed as
\[
y(t) = \frac{1}{2\pi} \int_0^{2\pi} a(\theta, \phi)s(\theta, t)d\theta + n(t),
\]
(5)
where \( s(\theta, t) \) is the distributed source density and, as usual, is temporally and spatially uncorrelated with \( n(t) \). Note that the case \( s(\theta, t) = \sum_{k=1}^L s_k(t)\delta(\theta - \theta_k) \) represents \( L \) point sources.

In this paper, as an extension of the investigation in [14], we consider the estimation of the azimuth and elevation angles under a distributed signal source model. Distributed sources are described by a distributed source density or a directionality which indicates the amount of source power coming from each direction as a continuum.

Denoting the 3-dimensional distributed source density by \( s(\theta, \phi, t) \), we have
\[
s(\theta, \phi, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} g_{mn}(t) e^{-jm\theta} e^{-jn\phi},
\]
(6)
where \( g_{mn}(t) \) is a random variable with \( E[g_{mn}(t)] = 0 \) and \( E[g_{mn}(t)g_{m'n}(t)] = \gamma_{mnkt} \). The expansion (6) is possible since \( s(\theta, \phi, t) \) is a periodic function of \( \theta \) and \( \phi \).

Then we have the covariance function \( R_s \) of the signal source as
\[
R_s(\theta, \phi, \theta', \phi') = E[s(\theta, \phi, t)s^*(\theta', \phi', t)]
\]
\[
= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \gamma_{mnkt} e^{-j(m\theta - km')} e^{-j(n\phi - ln')}.
\]
(7)

When the signal sources are point sources, i.e., when \( s(\theta, \phi, t) = \sum_{k=1}^L s_k(t)\delta(\theta - \theta_k)\delta(\phi - \phi_k) \), the covariance function \( R_s \) has peaks at the DOAs and the magnitudes are theoretically infinite. On the other hand, when the signal source density is distributed, the covariance function takes on large values around the center directions.

Under this distributed source density formulation, the output \( y(t) \) of an array can be expressed as
\[
y(t) = \frac{1}{4\pi^2} \int_0^{\pi} \int_0^{2\pi} a(\theta, \phi)s(\theta, \phi, t)d\theta d\phi + n(t),
\]
(8)
where \( n(t) \) is temporally and spatially uncorrelated with \( s(\theta, \phi, t) \). The expression (8) is a generalization of (4) and an extension of (5). If the \( L \) signal sources are impulse-like so that
\[
s(\theta, \phi, t) = 4\pi^2 \sum_{k=1}^L s_k(t)\delta(\theta - \theta_k)\delta(\phi - \phi_k),
\]
(9)
then we obtain the two-dimensional point source model of (4) from (8).

2.2 The Parametric Distributed Source Model

As we can see in (8), it is very difficult to proceed further for arbitrary 3-dimensional distributed source density \( s(\theta, \phi, t) \), unless some restrictions are imposed on the characteristics of \( s(\theta, \phi, t) \). In this paper, to obtain specific and concrete results, we will consider only a class of the distributed sources.

Let \( g_{mn}(t) = \sum_{k=1}^L s_k(t)\rho_k^m e^{im\theta_k} \eta_k^ne^{jn\phi_k} \), where \( \theta_k \) and \( \phi_k \) are the center angles (which will be called the DOAs in this paper) and \( \rho_k \) and \( \eta_k \) are the dispersion parameters (DPs) with \( 0 < \rho_k, \eta_k < 1 \), \( 0 \leq \theta_k < 2\pi \), and \( 0 < \phi_k < \pi \). Note that in [13] and [14], similar restrictions were also imposed on to get physical and analytic results to any degree.

We will call \( s_k(\theta, \phi, t) = s_k(t)\sum_{m=0}^{\infty} \rho_k^m e^{-jm(\theta - \theta_k)} \sum_{n=0}^{\infty} \eta_k^ne^{-jn(\phi - \phi_k)} \) defined with the four parameters \((\theta_k, \rho_k, \phi_k, \eta_k)\) a (3-dimensional) parametric source that is distributed around the center angles \((\theta_k, \phi_k)\). Note that \( s(\theta, \phi, t) = \sum_{k=1}^L s_k(\theta, \phi, t) \). Thus, a parametric source is characterized by the four parameters, two DOAs \( \theta_k \) and \( \phi_k \) (representing the center directions for the horizontal and vertical reference axes), and two DPs \( \rho_k \) and \( \eta_k \) (representing the degrees of dispersion for the horizontal and vertical reference axes). Under this parametric source model, the goal is to estimate \( \theta_k, \rho_k, \phi_k, \) and \( \eta_k \), for \( k = 1, 2, \cdots, L \).
Now, the distributed source density composed of $L$ parametric sources can be expressed as

$$s(\theta, \phi, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} g_{mn}(t) e^{-j m \theta} e^{-j n \phi}$$

$$= \sum_{k=1}^{L} s_k(t) \sum_{m=0}^{\infty} \rho_k^m e^{-j m(\theta - \theta_k)} \sum_{n=0}^{\infty} \eta_k^n e^{-j n(\phi - \phi_k)}$$

$$= \sum_{k=1}^{L} s_k(t) I_k(\theta, \phi),$$

(10)

where $I_k(\theta, \phi) = I(\theta, \phi; \theta_k, \rho_k, \phi_k, \eta_k) = 1/[(1 - \rho_k e^{-j(\theta - \theta_k)})(1 - \eta_k e^{-j(\phi - \phi_k)})]$ will be called the intensity function of the $k$th source, with $I(\theta, \phi; \theta', \rho', \phi', \eta') = 1/[(1 - \rho' e^{-j(\theta - \theta')})(1 - \eta' e^{-j(\phi - \phi')})]$.

The array output of the parametric sources with density (10) can be written as

$$y(t) = \sum_{k=1}^{L} \frac{s_k(t)}{4\pi^2} \int_{-\pi}^{\pi} \int_{0}^{2\pi} \mathbf{a}(\theta, \phi) I_k(\theta, \phi) d\theta d\phi + \mathbf{n}(t).$$

(11)

It is noteworthy that the intensity function $I_k$ defined on the region $\{(\theta, \phi): 0 \leq \theta < 2\pi, 0 \leq \phi < \pi\}$ is the intensity of the $k$th source "seen at" the array: for a differential area $d\theta d\phi$, therefore, the power received by the array is $I_k(\theta, \phi) d\theta d\phi$. The array output vector in (11) can be obtained by the Cauchy integration since $\mathbf{a}(\theta, \phi)$ is specified, which depends on the array structure. Note that (11) becomes (4) when all $\rho_k, \eta_k \to 1$, since

$$y(t) = \sum_{k=1}^{L} \frac{s_k(t)}{4\pi^2} \int_{-\pi}^{\pi} \int_{0}^{2\pi} \mathbf{a}(\theta, \phi) I_k(\theta, \phi) d\theta d\phi + \mathbf{n}(t)$$

$$= \sum_{k=1}^{L} \frac{s_k(t)}{2\pi} \int_{0}^{2\pi} \mathbf{a}(\theta_k, \frac{\phi}{2}) \frac{d\lambda}{2} + \mathbf{n}(t)$$

$$= \sum_{k=1}^{L} s_k(t) \mathbf{a}(\theta_k, \phi_k) + \mathbf{n}(t).$$

(12)

When the analysis frame size, or the number of snapshots, for the estimation is $N$, the sample covariance function of the array output can be obtained as, for example,

$$\hat{R}_y = \frac{1}{N} Y Y^H,$$

(13)

where $Y = [y(t_1), \ldots, y(t_N)]$ is an $M \times N$ matrix. We then find the null spectrum by using, for example, the MUSIC-like eigen-decomposition method with the sample covariance function of the output. In other words, the set of the four parameters, $\theta_k, \rho_k, \phi_k, \eta_k$, $k = 1, 2, \ldots, L$, can be estimated as

$$\hat{\theta}, \hat{\rho}, \hat{\phi}, \hat{\eta} = \arg\max_{\theta, \rho, \phi, \eta} V(\theta, \rho, \phi, \eta),$$

(14)

where $\hat{\theta} = [\hat{\theta}_1, \ldots, \hat{\theta}_L], \hat{\rho} = [\hat{\rho}_1, \ldots, \hat{\rho}_L], \hat{\phi} = [\hat{\phi}_1, \ldots, \hat{\phi}_L], \text{ and } \hat{\eta} = [\hat{\eta}_1, \ldots, \hat{\eta}_L]$ are the estimated vectors, and $V$ is a null spectrum which depends on the array used in the estimation.

3. DOA Estimation with a Uniform Circular Array

In this section, we consider estimation of the DOA and DP with the covariance matrices obtained from a circular array. Consider a uniform circular array consisting of $M$ identical omnidirectional sensors evenly spaced on a ring of radius $R$ on the XY plane with its center at the origin of the Cartesian coordinate system, as shown in Fig. 1. The position vector $\mathbf{r}_k$ of the $k$th sensor can be expressed as

$$\mathbf{r}_k = (R \cos \alpha_k, R \sin \alpha_k, 0), \quad k = 1, 2, \ldots, M,$$

(15)

where $\alpha_k = 2\pi(k - 1)/M$ is the angular position of the $k$th sensor measured counter-clockwise from the positive X-axis in radian.

From (11), the array output can be written as

$$y(t) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{0}^{2\pi} \mathbf{a}_c(\theta, \phi) s(\theta, \phi, t) d\theta d\phi + \mathbf{n}(t)$$

$$= \sum_{k=1}^{L} s_k(t) \mathbf{b}_{c,k} + \mathbf{n}(t),$$

(16)

where $\mathbf{a}_c(\theta, \phi)$ is the steering vector of the circular array (see Appendix) and $\mathbf{b}_{c,k} = \mathbf{b}_c(\theta_k, \rho_k, \phi_k, \eta_k), k = 1, 2, \ldots, L$, with

$$\mathbf{b}_c(\theta, \rho, \phi, \eta)$$

$$= \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{0}^{2\pi} a_c(\xi, \zeta) d\xi d\zeta$$

$$= \frac{1}{4\pi^2} \int_{0}^{2\pi} \int_{0}^{2\pi} \frac{\mathbf{a}_c(\xi, \zeta) d\xi d\zeta}{(1 - \rho e^{-j(\xi - \theta)})(1 - \eta e^{-j(\zeta - \phi)})}.$$

(17)

It is easy to see that $\mathbf{b}_c$ is an averaged version of the

![Fig. 1 A uniform circular array.](image)
steering vector \( a_c \). From now on, we will use \( \psi_{k} \) and \( \psi_{i,k} \) to denote \( \psi_k e^{j\theta} \) and \( \psi_k e^{j\phi} \), respectively, for notational convenience. The \( i \)th element of the column vector \( b_{c,k} \) is, using the Cauchy integral formula,

\[
b_{c,k,i} = \frac{1}{4\pi^2} \int_0^{2\pi} e^{j\frac{2\pi}{\lambda_c} (\cos \phi \psi_{k} e^{j\theta} + \sin \phi \psi_{k} e^{j\phi})} d\phi d\theta
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\psi_{i,k} e^{-j\phi}} \frac{1}{1 - \psi_{k} e^{-j\phi}} d\phi
\]

\[
= \frac{1}{2\pi} \int_c \frac{e^{j\phi/2}(x + \frac{1}{2})(\psi_k + \frac{1}{\psi_k})}{1 - \psi_k e^{-j\phi}} dz
\]

\[
= \exp \left[ \frac{\pi R}{2\lambda_c} \left( \psi_{i,k} + \frac{1}{\psi_{i,k}} \right) \right]
\]

\[
\cdot \left( \psi_k e^{j\phi} + \psi_k e^{-j\phi} \right)
\]  

(18)

Since the source covariance function of \( s(\theta, \phi, t) \) in the parametric source model is

\[
R_s(\theta, \phi, t, t') = E[s(\theta, \phi, t)s^*(\theta, \phi', t)]
\]

\[
= \sum_{m=1}^{L} \sum_{n=1}^{L} [p_{mn} \cdot I_m(\theta, \phi) \cdot I_n^*(\theta, \phi')]
\]

(19)

the covariance matrix \( R_{c,y} \) of the uniform circular array output is given by

\[
R_{c,y} = B_c \Delta B_c^H + \sigma^2 I
\]

(20)

where \( \Delta_{mn} = p_{mn} = E[s_m(t)s_n^*(t)] \) for \( m, n = 1, 2, \cdots, L \), and \( B_c = [b_{c,1}, b_{c,2}, \cdots, b_{c,L}] \).

By the eigendecomposition of \( R_{c,y} \) we can obtain the signal subspace \( S_c = [e_{c,1}, e_{c,2}, \cdots, e_{c,L}] \) and the noise subspace \( G_c = [e_{c,L+1}, e_{c,L+2}, \cdots, e_{c,M}] \), where \( e_{c,k} \) is the eigenvector corresponding to the \( k \)th largest eigenvalue \( \lambda_{c,k} \) of \( R_{c,y} \) with \( \lambda_{c,1} > \lambda_{c,2} > \cdots > \lambda_{c,L} = \lambda_{c,L+1} = \cdots = \lambda_{c,M} \). Since \( \text{span}(e_{c,L+1}, e_{c,L+2}, \cdots, e_{c,M}) \) is orthogonal to \( \text{span}(e_{c,1}, e_{c,2}, \cdots, e_{c,L}) \), the parameters \( (\theta_k, \rho_k, \phi_k, \eta_k) \), \( k = 1, 2, \cdots, L \), can be estimated using the following orthogonality property:

\[
b_c^H(\theta, \rho, \phi, \eta)G_c = 0 \quad i f f \quad (\theta, \rho, \phi, \eta) \in \{(\theta_1, \rho_1, \phi_1, \eta_1), \cdots, (\theta_L, \rho_L, \phi_L, \eta_L)\}
\]

(21)

In practice, we first obtain the MUSIC null spectrum

\[
f_{MU}(\theta, \rho, \phi, \eta)
\]

\[
= \frac{|b_c^H(\theta, \rho, \phi, \eta)||G_c^H G_c b_c(\theta, \rho, \phi, \eta)||^2}{|b_c^H(\theta, \rho, \phi, \eta)||G_c^H G_c b_c(\theta, \rho, \phi, \eta)|^2}
\]

(22)

where \( G_c \) is the noise subspace of \( \hat{R}_{c,y} \), an estimate of \( R_{c,y} \). Then the parameters \( (\theta_k, \rho_k, \phi_k, \eta_k) \), \( k = 1, 2, \cdots, L \), can be estimated, for example, with the following procedure:

\[
(\hat{\theta}_k, \hat{\rho}_k, \hat{\phi}_k, \hat{\eta}_k) = \arg \max_{(\theta, \rho, \phi, \eta)} f_{MU}(\theta, \rho, \phi, \eta)
\]

for \( k = 1, 2, \cdots, L \).

(23)

A direct estimation of (23), however, requires heavy burden of calculation. Henceforth, we will in this paper consider two-step two-procedures based on 2-dimensional maximization methods.

(Procedure 1) First, we obtain the azimuth null spectrum

\[
f_{A}(\theta, \rho) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^1 f_{MU}(\theta, \rho, \phi, \eta) d\phi d\eta\]

(24)

to estimate the azimuth parameters \( (\theta_k, \rho_k) \) as

\[
(\hat{\theta}_k, \hat{\rho}_k) = \arg \max_{(\theta, \rho)} f_{A}(\theta, \rho)
\]

for \( k = 1, 2, \cdots, L \).

(25)

The elevation parameters \( (\phi_k, \eta_k) \) can then be estimated as

\[
(\hat{\phi}_k, \hat{\eta}_k) = \arg \max_{(\phi, \eta)} f_{MU}(\hat{\theta}_k, \hat{\rho}_k, \phi, \eta)
\]

for \( k = 1, 2, \cdots, L \).

(26)

(Procedure 2) Obtain the DOA null spectrum

\[
f_{D}(\theta, \phi) = \int_0^1 \int_0^1 f_{MU}(\theta, \rho, \phi, \eta) d\rho d\eta
\]

(27)

from which the simultaneous estimation of \( (\theta_k, \phi_k) \), \( k = 1, 2, \cdots, L \), is accomplished as

\[
(\hat{\theta}_k, \hat{\phi}_k) = \arg \max_{(\theta, \phi)} f_D(\theta, \phi).
\]

(28)

Then the DPs can be estimated as

\[
(\hat{\rho}_k, \hat{\eta}_k) = \arg \max_{(\rho, \eta)} f_{MU}(\hat{\theta}_k, \hat{\rho}_k, \phi_k, \eta_k)
\]

for \( k = 1, 2, \cdots, L \).

(29)

We have \( L \) sets of \( 4 \) parameters to be estimated. Among the six possible two-step procedures, it is easy to see that the two 'reversed' ones of Procedures 1 and 2 would be the same as Procedures 1 and 2, respectively, in terms of performance. The other two procedures, in which the azimuth (elevation) DOA and the elevation (azimuth) DP are estimated at a time, are obviously unreasonable.

Since we consider the fractional separable and unimodal sources, for which the intensity function is given by \( I_k(\theta, \phi) \), we can expect that the performance of Procedures 1 and 2 will be almost the same as we shall see in Example 2 below: we will thus consider Procedure 2 only in Simulation Results.
4. Some Examples

In this section, to see the developments in the previous sections more explicitly, we will consider some estimation examples and simulation results with a circular array shown in Fig. 1 when $L = 2$ and $M = 10$.

Consider the two equal-power uncorrelated sources with parameter value sets

$$(\theta_1, r_1, \phi_1, \eta_1) = (30^\circ, 0.9, 25^\circ, 0.8)$$

and

$$(\theta_2, r_2, \phi_2, \eta_2) = (40^\circ, 0.7, 40^\circ, 0.6),$$

for which the intensity functions $I_1(\theta, \phi)$ and $I_2(\theta, \phi)$ are shown in Figs. 2(a) and (b), respectively. Here the $SNR$ is defined as $10\log [E\{|s_1(t)|^2\}/\sigma^2] = 10\log [p_{11}/\sigma^2]$ (dB).

**Example 1:**

To see the necessity of the parametric source model, we obtained the conventional MUSIC null spectrum

$$f_P(\theta, \phi) = \frac{||a_c(\theta, \phi)||^2}{a^H(\theta, \phi) G_c G^H_c a_c(\theta, \phi)}$$

(30)

under the point source assumption at $SNR = 15$ dB, assuming that $\hat{G}_c = G_c$. Figure 3(a) shows the null spectrum (30) and the contour is shown in Fig. 3(b). Obviously, only one maximum point can be found, and the other source cannot be located. This example shows that the MUSIC-based estimator may fail when depending upon use of a point source assumption in the parametric source model.

**Example 2:**

In this example, let us show that Procedures 1 and 2 would produce (almost) the same results. Assume that $\hat{G}_c = G_c$, which is equivalent to the assumption of $N \rightarrow \infty$, and that $SNR = 15$ dB.

[Procedure 1] We estimate the two-sets of four parameters with the maximization processes (25) and (26) using (24) and (22). We first estimate the azimuth parameter sets $(\theta_1, r_1)$ and $(\theta_2, r_2)$ with (25). Next, using the set $(\hat{\theta}_1, \hat{r}_1)$ in (22), we estimate $(\hat{\phi}_1, \eta_1)$. Finally, we use $(\hat{\theta}_2, \hat{r}_2)$ to estimate $(\hat{\phi}_2, \eta_2)$ with (26). Figures 4.1(a)–(f) show the null spectra and their contours obtained from Procedure 1.

[Procedure 2] Consider the null spectra (27) and (22). Figure 4.2(a) shows the null spectrum $f_P(\theta, \phi)$ of (27). In this figure, we can observe two peaks in the three-dimensional space. Figure 4.2(b) shows the contour of $f_P(\theta, \phi)$, with which the DOAs can be estimated to be $(\hat{\theta}_1, \phi_1) = (30^\circ, 25^\circ)$ and $(\hat{\theta}_2, \phi_2) = (40^\circ, 40^\circ)$ by the maximization procedure (28).
Fig. 4.1(a) The null spectrum $f_A(\theta, \rho)$ with $G_e = G_c$.

Fig. 4.1(b) The contour of Fig. 4.1(a).

Fig. 4.1(c) The null spectrum $f_{MU}(\hat{\theta}_1, \beta_1, \phi, \eta)$ with $G_e = G_c$.

Fig. 4.1(d) The contour of Fig. 4.1(c).

Fig. 4.1(e) The null spectrum $f_{MU}(\hat{\theta}_2, \beta_2, \phi, \eta)$ with $G_e = G_c$.

Fig. 4.1(f) The contour of Fig. 4.1(e).
Fig. 4.2(a) The null spectrum $f_D(\theta, \phi)$ with $\hat{G}_c = G_c$.

Fig. 4.2(b) The contour of Fig. 4.2(a).

Fig. 4.2(c) The null spectrum $f_{MU}(\hat{\theta}_1, \rho, \hat{\phi}_1, \eta)$ with $\hat{G}_c = G_c$.

Fig. 4.2(d) The contour of Fig. 4.2(c).

Fig. 4.2(e) The null spectrum $f_{MU}(\hat{\theta}_2, \rho, \hat{\phi}_2, \eta)$ with $\hat{G}_c = G_c$.

Fig. 4.2(f) The contour of Fig. 4.2(e).
Fig. 5(a) The null spectrum $f_{D}(\theta, \phi)$ when $SNR = 20$ dB.

Fig. 5(b) The contour of Fig. 5(a).

Fig. 5(c) The null spectrum $f_{MU}(\hat{\theta}_1, \rho, \hat{\phi}_1, \eta)$ when $SNR = 20$ dB.

Fig. 5(d) The contour of Fig. 5(c).

Fig. 5(e) The contour of Fig. 5(e).
Fig. 6(a) The null spectrum $f_D(\theta, \phi)$ when $SNR = 10\, \text{dB}$.

Fig. 6(b) The contour of Fig. 6(a).

Fig. 6(c) The null spectrum $f_{MU}(\hat{\theta}_2, \rho, \hat{\phi}_2, \eta)$ when $SNR = 10\, \text{dB}$.

Fig. 6(d) The contour of Fig. 6(c).

Fig. 6(e) The null spectrum $f_{MU}(\hat{\theta}_2, \rho, \hat{\phi}_2, \eta)$ when $SNR = 10\, \text{dB}$.

Fig. 6(f) The contour of Fig. 6(e).
Next, we use the values \((\hat{\phi}_1, \hat{\phi}_2) = (30^\circ, 25^\circ)\) to obtain \((\hat{\rho}_1, \hat{\eta}_1)\) from (29), as shown in Figs. 4.2(c) and (d). Similarly, to estimate the DPs \((\rho_2, \eta_2)\) of the second source, we use the values \((40^\circ, 40^\circ)\) of \((\hat{\phi}_2, \hat{\phi}_2)\) in (29): the null spectrum \(f_{MU}(\hat{\theta}_2, \rho_2, \hat{\phi}_2, \eta_2)\) is shown in Fig. 4.2(c), and Fig. 4.2(f) shows the contour, from which we can estimate \((\rho_2, \eta_2)\).

**Simulation Results:**

We set the number of samples, \(N = 100\). The noise subspace \(\mathcal{G}_c\) is obtained through the eigen-decomposition of the sample covariance function \(\mathcal{R}_{c,c} = \frac{1}{N} Y_c Y_c^H\) obtained based on random number generation of the two vectors \(s(t) = [s_1(t), s_2(t), \ldots, s_L(t)]^T\) and \(n(t) = [n_1(t), n_2(t), \ldots, n_M(t)]^T\), where \(Y_c = \left[ y_c(t_1) \quad y_c(t_2) \quad \cdots \quad y_c(t_N) \right]\) is the \(M \times N\) circular array output matrix. In the simulations, we estimate the 2 sets of four parameters from the implementation of Procedure 2 when the \(SNR = 20, 15, 10,\) and 5 dB. When the \(SNR = 20\) dB, Figs. 5(a) and (b) show the null spectrum (27) and its contour, respectively, used for the estimation of the DOAs. Figures 5(c) and (d) show the implementation of (29) for the estimation of the DPs of the first source. The implementation of (29) for the estimation of the DPs of the second source is shown in Figs. 5(e) and (f).

When the \(SNR = 15, 10,\) and 5 dB, we obtained similar but slightly inferior results as is partially shown in Figs. 6(a)–(f). Naturally, as the \(SNR\) decreases the performance gets inferior.

### 5. Conclusion

In some real environment, signal source localization should be performed based on a distributed model in the 3-dimensional space. When the signal sources are not point sources, but dispersed over an area, we cannot directly use the well-known direction of arrival estimation methods, because these methods are established based on the point source assumption.

In this paper, we proposed a 3-dimensional distributed signal source model which is represented with the center angles and degrees of dispersion, and investigated some eigenstructure-based algorithms to locate the two-dimensional distributed signal sources. We showed that the performance of the method was acceptable when the signal sources were distributed while the conventional method based on the point source model failed to correctly estimate the DOAs.

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### References


### Appendix

From Fig. 1, the steering vector of a uniform circular array is

\[
\alpha_c(\theta, \phi) = \begin{bmatrix}
\alpha_{c1} \\
\alpha_{c2} \\
\vdots \\
\alpha_{cN}
\end{bmatrix} = \begin{bmatrix}
e^{j2\pi R \cos \phi \cos (\theta - \alpha_1)} \\
e^{j2\pi R \cos \phi \cos (\theta - \alpha_2)} \\
\vdots \\
e^{j2\pi R \cos \phi \cos (\theta - \alpha_M)}
\end{bmatrix}, \tag{A.1}
\]
where $R$ is the radius of the uniform circular array and $\lambda_c$ is the wavelength of the signal.

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