Scattering cross sections on different eccentricities of EMS with region have been presented. Also the convergence of the modal plane wave incidence. Numerical results for EMS in the resonance radome, or resonator with spherical boundary. If the innermost core regions were expanded by the spherical vector wave functions, and then the addition theorem for the spherical coordinate system was given as (a) (O,0,0), (b) (0.1,0.1,0.1), (c) (-0.1,0.1,0.1), and (d) (-0.1, -0.1, -0.1). and (e) (-0.1, 0.1, -0.1).

V. CONCLUSION

An exact series solution for scattering from EMS has been found when the innermost core was a conducting sphere. EM fields in all regions were expanded by the spherical vector wave functions, and then the addition theorem for the spherical coordinate system was used to apply the boundary conditions. A system of linear equations was derived from the boundary conditions. By solving this equation, the far-zone scattered field patterns have been evaluated for a uniform plane wave incidence. Numerical results for EMS in the resonance region have been presented. Also the convergence of the modal solutions have been investigated with various dielectric distributions, eccentricities, and dimensions. We found that the convergency of the solution only depends on the dimension of the scatterer.

From these results, we can predict fields in the dielectric lens, radome, or resonator with spherical boundary. If the innermost core is dielectric, an exact series solution can be found by a slight modification of the boundary condition on the core.

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Efficient Kernel Calculation of Cylindrical Antennas

Seong-Ook Park and Constantine A. Balanis

Abstract—This paper presents a technique for evaluating analytically and without limitation the singular part of the kernel integral of cylindrical wires due to uniform current distribution. This approach uses the exact Green's function expression in cylindrical coordinates. The formula of the singular part converges rapidly and illustrates its usefulness for kernel calculations without loss of accuracy.

I. INTRODUCTION

The practical numerical calculation of the thin-wire kernel of cylindrical wires typically relies on the static or singular part of the double integral 1/R, on the surface of the cylinder [1]-[6]. Butler [4] evaluated the integral with the series form valid only for Δ/2a > 1. Another approach evaluates this problem by approximating a cylindrically curved subsection in the neighborhood of the singularity by a flat rectangular patch [5]. An exact expression for the kernel integration in cylindrical antennas has been obtained recently by Wang [7] and Werner [8]. The double integral 1/R, on the cylindrical surface, however, is not available in the literature.

In this paper, a closed-form expression for the singular part due to uniform current distribution is presented. Although the exact expression is available, this method can be used to evaluate efficiently and accurately the matrix elements in a moment-method solution of thin wires.

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The authors are with the Department of Electrical Engineering, Telecommunication Research Center, Arizona State University, Tempe, AZ 85287-7206 USA.

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II. KERNEL CALCULATION

Field expressions on the surface of a wire from Maxwell’s equation and boundary conditions are written as [9], [10]

\[
\begin{align*}
-E_i &= -j\omega A_i - \frac{\partial \phi}{\partial l} \\
A_i &= \frac{\mu}{4\pi} \int \int J_s e^{-j|\mathbf{r} - \mathbf{r}'|} ds' \\
\phi &= \frac{1}{4\pi e_0} \int \int \rho_s e^{-j|\mathbf{r} - \mathbf{r}'|} ds' \\
\rho_s &= \frac{1}{j\omega} \frac{J_s dl}{l}
\end{align*}
\]
(1a) (1b) (1c) (1d)

where \( l \) is the length along the wire axis, and \( J_s \) and \( \rho_s \) are the surface current and charge densities, respectively.

Assuming the wire is excited by a rotationally symmetric tangential electric field \( E_i \), the only nonvanishing components are the axial electric field and axial surface current density \( J_s \), which can be expressed as \( J_s = I/(2\pi a) \). The current \( I \) is assumed to flow only in the direction of the wire axis.

The moment method is used to discretize (1) into a matrix equation of the form

\[
[Z_m][I_n] = [V_m]
\]
(2)

\([I_n]\) and \([V_m]\) are column matrices with \( n = 1, 2, 3, \ldots, N \) and \( m = 1, 2, 3, \ldots, N \) where \( N \) is equal to the number of current pulses. \([V_m]\) represents an applied voltage vector while \([I_n]\) represents the unknown current vector. The current elements are approximated by filaments of uniform current and delta function charge densities at both ends of the sub-dipole.

The basic integral in the impedance matrix \([Z_m]\) for evaluating the magnetic vector and electric scalar potentials due to z-directed uniform currents is of the form

\[
\Im \left[ r - r' \right] = \int_0^\Delta dw \frac{\exp(-j\beta R_z)}{R_z} d\phi' dz'
\]
(3a)

where

\[ R_z = \sqrt{(z - z')^2 + 4a^2 \sin^2((\phi - \phi')/2)} \]

In (3a), \( |r - r'| \) is the distance from the source point to the observation point on the wire surface. We regard the one segment length as \( 2\Delta \) for the convenience of the kernel calculation as in [4]. The integrand of (3) is singular at \( z = z', \phi = \phi' \). For convenience, and using symmetry, the singularity is translated to \( z = 0, \phi = 0, \rho = a \). To solve (3), it is convenient to write it as [4]

\[
\Im = \Im_0 + \Im_1 - \Im_2 + \Im_3
\]
(4)

\[
\Im_0 = \frac{1}{2\pi} \int_0^\Delta dw \frac{\exp(-j\beta R_z)}{R_z} d\phi' dz'
\]
(4a)

\[
\Im_1 = \frac{1}{2\pi} \int_0^\Delta dw \frac{\exp(-j\beta R_z - 1)}{R_z} d\phi' dz'
\]
(4b)

\[
\Im_2 = \frac{1}{2\pi} \int_0^\Delta dw \frac{\exp(-j\beta R_z - 1)}{R_z} d\phi' dz'
\]
(4c)

\[
\rho_s = \frac{1}{j\omega} \frac{J_s dl}{l}
\]
(1d)

where the first term of the integrand of (4) contains the singularity and the second term is a slow, i.e., varying function presenting no difficulties to numerical calculations.

To evaluate the double integral of (4a) on the surface of the cylinder, first we investigate the static Green’s function in cylindrical coordinates. The \( 1/|r - r'| \) value in spherical coordinates can be expressed as a series of products of harmonic functions in cylindrical coordinates like [11]

\[
\frac{1}{|r - r'|} = \frac{2}{\pi} \int_0^\infty \cos(k(z - z')) I_0(k(p)) K_0(k(p')) dp
\]
(5)

where \( I_0(k(p)) \) and \( K_0(k(p)) \) are modified Bessel functions of the first and second kind, respectively.

If the source point is located on \( 0 < z' < \Delta, 0 < \phi' < 2\pi, \) and \( \rho' = a \), only the first term in (5) is needed with the integration of \( \phi' \) from 0 to \( 2\pi \) at the observation point \( z = 0, \phi = 0, \rho = a \), or

\[
\Im_0 = \frac{1}{2\pi} \int_0^\Delta dw \frac{\exp(-j\beta R_z)}{R_z} d\phi' dz'
\]
(6)

After reversing the order of integration, (6) reduces to

\[
\Im_0 = \frac{2}{\pi} \int_0^\Delta dw \frac{\sin \Delta k}{k} K_0(k) I_0(k) dk.
\]
(7)

The modified Bessel function \( I_0(k) \) can be represented by the series expression (from formula 8.447.1 of [12])

\[
I_0(k) = \frac{1}{(n!)^2} \left( \frac{a^2}{4} \right)^n k^{2n-1}.
\]
(8)

Substituting (8) into (7) and interchanging the order of summation, (7) can be expressed as

\[
\Im_0 = \frac{2}{\pi} \sum_{n=0}^\infty \frac{1}{(n!)^2} \left( \frac{a^2}{4} \right)^n \int_0^\Delta k^{2n-1} \sin(\Delta k) K_0(k) dk.
\]
(9)

With the aid of the integration formula 6.699.3 of [12], (9) is written in series form as

\[
\Im_0 = \frac{1}{2\pi} \int_0^\Delta dw \frac{\exp(-j\beta R_z)}{R_z} d\phi' dz'
\]
(10)

\[
= \sum_{n=0}^\infty \frac{1}{(n!)^2} \left( \frac{a^2}{4} \right)^n k^{2n+1} \frac{\sin(\Delta k)}{\Delta k} \left( \frac{\sin(\Delta k)}{\Delta k} \right)^{2p} \left( \frac{\Delta k}{a} \right)^2 < 1
\]
(10a)

which is valid only for \( |\Delta/a| < 1 \).
Although the evaluation of (10) is exact, it contains many series which tend to be somewhat cumbersome in numerical evaluation and it is valid only for \(|\Delta/a| < 1\). Therefore, an alternative and more efficient method of evaluating (9) can be derived by integrating term by term. Doing this, (9) can be written as

\[
\Theta_0 = \frac{2}{\pi} \int_0^\infty \sin(\Delta k)K_0(ak) \times \left\{ \frac{1}{k} + \left(\frac{a^2}{4}\right)k + \frac{1}{(2!)^2} \left(\frac{a^2}{4}\right)^2 k^3 + \frac{1}{(3!)^2} \left(\frac{a^2}{4}\right)^3 k^3 + \cdots \right\} \, dk = A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + \cdots
\]

(11)

The integrals for the \(A_0\) and \(A_1\) terms can be easily obtained analytically from formulas 6.699.3 and 6.691 of [12], or

\[
A_0 = \frac{2}{\pi} \int_0^\infty \frac{\sin(\Delta k)}{k} K_0(ak) \, dk = \int_0^\Delta \frac{dz'}{\sqrt{z^2 + a^2}} = \ln\left(\frac{\Delta}{a} + \frac{1}{\left(\frac{\Delta}{a}\right)^2}\right)
\]

(12)

\[
A_1 = \frac{2}{\pi} \int_0^\infty \frac{a^2}{4} k \sin(\Delta k) K_0(ak) \, dk = \frac{1}{4} \left(\frac{\Delta}{a}\right)^2 \frac{1}{\sqrt{\frac{a^2}{4} + 1}}
\]

(13)

Closed-form solutions can be obtained for the remaining terms \((A_2, A_3, \cdots)\) with the recurrence relation of the modified Bessel function and the following formulas (8.486.10, 6.699.11 of [12])

\[
K_{n+1}(z) = \frac{2\nu}{z} K_n(z) + K_{n-1}(z)
\]

(14)

\[
\int k^{n+1} \sin(\Delta k) K_n(ak) \, dk = \sqrt{\pi} (2a)^n \Gamma\left(\frac{3}{2} + \nu\right) \times (a^2 + \Delta^2)^{-\frac{3}{2} - \nu}.
\]

(15)

Substituting (15) in (14), and after some manipulations, it yields

\[
A_2 = \frac{2}{\pi} \frac{1}{\left(2!ight)^2} \frac{a^2}{4} \int_0^\infty k^3 K_0(ak) \sin(\Delta k) \, dk = -\frac{5!!}{4^2 \left(2!ight)^2} \left(\frac{\Delta}{a}\right) \left(1 + \left(\frac{\Delta}{a}\right)^2\right)^{\frac{3}{2} - \frac{1}{2}}
\]

- \left(\frac{2}{\left(3!!\right)^2}\right) \left(\frac{\Delta}{a}\right)^2 \left(1 + \left(\frac{\Delta}{a}\right)^2\right)^{\frac{3}{2} - \frac{1}{2}}
\]

A_3 = \frac{2}{\pi} \frac{1}{\left(3!ight)^2} \frac{a^2}{4} \int_0^\infty k^3 K_0(ak) \sin(\Delta k) \, dk
\]

(16)

\[
= \frac{9!!}{4^3 \left(3!ight)^2} \left(\frac{\Delta}{a}\right) \left(1 + \left(\frac{\Delta}{a}\right)^2\right)^{\frac{3}{2} - \frac{1}{2}}
\]

- \left(\frac{8}{\left(5!!\right)^2}\right) \left(\frac{\Delta}{a}\right)^2 \left(1 + \left(\frac{\Delta}{a}\right)^2\right)^{\frac{3}{2} - \frac{1}{2}} + \left(\frac{8}{\left(6!!\right)^2}\right) \left(\frac{\Delta}{a}\right)^3 \left(1 + \left(\frac{\Delta}{a}\right)^2\right)^{\frac{3}{2} - \frac{1}{2}}
\]

(17)

The values obtained using the formulas of this paper are compared with MININEC data and the value taken from King-Middleton \(83.6 + j41.34\) in which the antenna radius \(a\) and length \(L\) are 0.001588X and \(\lambda/2\), respectively [13].

The remaining terms can be found using a similar procedure but usually have negligible values.

### III. Computations and Comparisons

The utilization of the derived formulas in moment method is illustrated in Fig. 1 which is a plot of (11) in the range of 0 < \(|\Delta/a| < 2.5\). Note that the dominant term of \(\Theta_0\) is \(A_0\).

Plots of the input impedance of a dipole antenna versus segment number are shown in Fig. 2. To show the effect of segment length-to-radius, the segment number ranges from 4 to 200, corresponding to \(|\Delta/a|\) from 39.35 to 0.7871. The values obtained using the formulas of this paper are compared with MININEC data and the value taken from King-Middleton (83.6 + j41.34 in which the antenna radius \(a\) and length \(L\) are 0.001588X and \(\lambda/2\), respectively [13]).
In evaluating the impedance matrix, the singularity occurs only in the real part, and therefore $S_0$ of (11) influences the real values. The imaginary value is affected by the imaginary part of $S_1$ in (4b). For the computation using the formulas derived in this paper, two cases were considered. For case 1 only the first term $A_0$ of (11) was used, while for case 2 only the first two terms, $A_0$ and $A_1$, were used. It is apparent when compared with the MININEC data that for $\Delta \alpha > 0.5$ (segment number 80) only the first term $A_0$ of (11) is needed for accurate evaluation of the real part of the impedance, while for $\Delta \alpha < 0.8$ (segment number 200) only the first two terms ($A_0$ and $A_1$) of (11) are needed for accurate evaluation of real part of the impedance. If $\Delta \alpha < 0.8$ (segment number greater than 200), additional terms in (11) may be needed. For most practical thin-wire antennas, however, $\Delta \alpha > 1$.

IV. CONCLUSION

The static double integral $1/R_a$ in kernel integral was evaluated by a closed and rapidly convergent series which has many practical numerical advantages. The computed data are compared with MININEC values and with a value from the King-Middleton. The results of this paper significantly improve the computational efficiency of the input impedance of thin-wire antennas.

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Simple and Accurate Formula for the Resonant Frequency of the Circular Microstrip Disk Antenna

Nirun Kumprasert and Wiwat Kiranon

Abstract—A simple algebraic formula for the resonant frequency of a circular microstrip disk antenna as a function of the effective radius and of the fringing capacitance has been derived which is valid for electrically-thick dielectric substrates. Accuracy of the theoretical results of resonance is compared with previous theories and measured data.

I. INTRODUCTION

Because of the narrow bandwidth of microstrip antennas, the accurate determination of their resonant frequency is important in the design of microstrip antennas. One of the techniques to increase the bandwidth is to increase the thickness of the dielectric substrate proportionately. Most of the design formulas for the resonant frequency, however, are accurate only for thin dielectric substrate, normally of the order of $h/\lambda_d \leq 0.02$ where $h$ is the thickness of the dielectric substrate and $\lambda_d$ is the wavelength in the dielectric substrate. In this paper, we present a simple and accurate design formula for the resonant frequency of an electrically-thick circular disk. The design formula is based on the fringing capacitance that is related to the edge extension of a circular disk.

II. CALCULATIONS

Fig. 1 shows the geometrical configuration of a circular microstrip disk antenna. A perfectly conducting circular disk is placed on the top of a dielectric substrate backed by a perfectly conducting ground plane. The disk has a physical radius $r$. The dielectric substrate has a relative dielectric constant of $\varepsilon_r$ and thickness $h$.

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