Abstract—A pilot emitting amplify-and-forward relay and hop-by-hop beamforming (HBF) scheme are proposed for two-hop relaying systems. The conventional beamforming (CBF) is performed with respect to an overall channel from a source node (SN) to a destination node (DN) through a relay node (RN) while the proposed HBF performs two independent beamformings, from the SN to the RN and from the RN to the DN. For beamforming, the SN and the DN require the channel state information (CSI) from the SN to the RN and from the RN to the DN, respectively. From our analytic and numerical results, it is shown that the bit-error-rate (BER) performance of the proposed HBF is identical to that of CBF with the perfect CSI. However, the proposed HBF can achieve a lower BER than the CBF method since the channel estimation performance of the HBF system is better than that of CBF, and this is verified by deriving and comparing the mean square errors of the channel estimations of HBF and CBF.

I. INTRODUCTION

Recently, various wireless relay systems, where a relay node (RN) retransmits the received signal from a source node (SN) to a destination node (DN), have been proposed to improve the system capacity as well as the expansion of the communication coverage [1]–[5]. There are two kinds of relays according to the retransmission methods: amplify-and-forward and decode-and-forward. As mentioned in [4], amplify-and-forward relays, which simply amplify and forward (retransmit) the received signal, outperform decode-and-forward relays, which decode, encode, and forward the signal under certain conditions with less processing burden on the relay. Also, relay systems can be classified by two different duplex methods: full-duplex and half-duplex. A full-duplex relay can transmit and receive the signal simultaneously while a half-duplex relay receives and transmits separately [1]; therefore, the latter one can be implemented more simply than the former one.

In this paper, a full-duplex amplify-and-forward relay having a single transmit/receive antenna is employed to the beamforming multiple-input multiple-output (MIMO) system, as shown in Fig. 1. Among techniques achieving array processing gain using multiple antennas, beamforming is a simple approach when the channel state information (CSI) is available at the transmitter [6]. Here, we consider two types of the beamforming: conventional beamforming (CBF) and the proposed hop-by-hop beamforming (HBF). CBF is performed with the effective overall channel between the SN and the DN through the RN. However, the proposed HBF performs two independent beamformings from the SN to the RN, termed SN-RN, and from the RN to the DN, termed RN-DN, sequentially. Emitting pilots at the RN is a key technique for implementing the proposed HBF method. The pilot emitting relay transmits the training sequences (i.e., pilot signals) at a proper time, such as a transmit transition gap or a receive transition gap in WiMax systems [7]. Then, due to the channel symmetry in time division duplex (TDD) systems, the estimated channel RN-SN can be used for transmit beamforming at the SN, and the estimated channel RS-DN can be used for receive beamforming at the DN, sequentially. The analytical and numerical results show that the performances of both beamformings are identical when there is no channel uncertainty. Also, we derive analytically...
and compare numerically the mean square errors (MSEs) of the channel estimation of the CBF and HBF systems. Then, it is shown that channel estimation performance of HBF is better than CBF; therefore, HBF can achieve a lower bit-error-rate (BER) than CBF.

Throughout this paper, $E[·]$, $tr(·)$, $(·)^T$, and $(·)^H$ denote expectation, trace, transpose, and Hermitian transpose, respectively, and $||·||_F$ and $||·||_2$ are the absolute value, Frobenius norm of matrix, and vector two-norm, respectively.

This paper is organized as follows. Section II presents the system and signal model, and the CBF and HBF methods are described and compared with perfect CSI in Section III. The performance analysis in a practical channel CSI estimation scenario is shown in IV, and BER performances are demonstrated through computer simulation in Section V. Finally, we provide conclusions in Section VI.

II. SYSTEM AND SIGNAL MODEL

We consider a two-hop wireless communication system consisting of the SN, RN, and DN, as shown in Fig. 1. The SN and DN have $N_s$ and $N_d$ antennas, respectively. An amplify-and-forward RN having a single transmit/receive antenna is assumed to forward the signal from the SN to the DN in a half-duplex manner, i.e., using different time resource. Assuming that the distance of SN-DN is much larger than that of RN-DN, we did not consider the direct link of SN-DN. The elements of every channel vectors are independently identically distributed (i.i.d) zero-mean complex Gaussian random variables with a unit variance.

When $x(k)$ denotes the transmitted signal at the SN, in which $E|x(k)|^2 = 0$ and $E||x(k)||^2 = 1$ and $\rho_s$ is the average signal energy received at the RN over one symbol period through the RN-RN link, the received signal at the RN can be written as

$$y_r(k) = \sqrt{\rho_s} h^H w x(k) + z_r(k),$$

where $k$ is the discrete symbol time index, $w \in \mathbb{C}^{N_s \times 1}$ stands for the beamforming weight vector, and $z_r(k)$ represents a zero-mean additive white Gaussian noise (AWGN) at the RN with a variance $\sigma_z^2$. After the RN retransmits the received signal from the SN to the DN, the signal model received by the DN can be written as follows:

$$y_d(k) = \alpha g y_r(k) + z_d(k) = \alpha \sqrt{\rho_s} g h^H w x(k) + \alpha g z_r(k) + z_d(k),$$

where $z_d(k) \in \mathbb{C}^{N_d \times 1}$ is an AWGN vector at the DN with a covariance of $\sigma_d^2 I$ and $\alpha$ is a fixed amplification factor of the RN. Here, the $\alpha$ can be expressed as follows [4]:

$$\alpha = \sqrt{\frac{\rho_r}{E||y_r(k)||^2}} = \sqrt{\frac{\rho_r}{\rho_s + \sigma_z^2}},$$

where $\rho_r$ is the average received signal energy at DN over one symbol period through the RN-DN link. Then, to detect transmitted signal $x(k)$, the DN combines the received signals by using the combining vector $b \in \mathbb{C}^{N_d \times 1}$ as follows:

$$r(k) = b^H y_d(k) = \bar{x}(k).$$

III. PROPOSED HBF SYSTEM AND PERFORMANCE ANALYSIS UNDER PERFECT CSI

In this section, we will introduce two types of beamforming MIMO systems shown in Fig. 1: CBF and HBF. The CBF system shown in Fig. 1(a) is similar to the conventional transmit beamforming MIMO systems without RN. In the CBF system, the SN performs beamforming with the overall channel matrix, which is multiplied by vector channel $g$ and $h^H$. In contrast to CBF, the proposed HBF system shown in Fig. 1(b) performs two independent beamformings. The first- and second-hop beamformings are performed sequentially with the vector channel $h^H$ and $g$, respectively. In this section, we consider symbol-by-symbol transmission so that the time index $k$ is henceforth omitted.

To compare signal-to-noise ratios (SNRs) of CBF and HBF precisely, the received signal in (1) can be rewritten as follows:

$$y_d = \alpha \sqrt{\rho_s} u \Sigma \Psi^H w x + \alpha g z_r + z_d = \alpha \sqrt{\rho_s} \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma & 0_{N_d-1,N_s-1} \\ 0_{N_s-1,N_d-1} & 0_{N_d-1,N_s-1} \end{bmatrix} \begin{bmatrix} \Psi^H \\ \Psi^H \end{bmatrix} w x + \alpha g z_r + z_d,$$

where $\sigma_{ab}$ represents $a$-by-$b$ zero vector or matrix. The first equality in (4) comes from the singular value decomposition (SVD) property, in which $gh^H = \Psi \Sigma \Psi^H = u_1 \sigma v^H_1$, where the diagonal matrix $\Sigma$ has only one real singular value $\sigma$ of the effective channel $gh^H$. Now, for CBF, the SN can obtain the right singular vector $v_1$ as the optimal transmit beamforming vector $w$ and the DN can obtain the left singular vector $u_1$ as the optimal receive beamforming vector $b$ in (3) [8]. Here, we assume that CSI $gh^H$ is known at both of the SN and DN. Consequently, the combined signal of the CBF can be written as

$$r = \alpha \sqrt{\rho_s} \sigma x + \alpha u_1^H g v_1 + u_1^H z_d$$

and the average received SNR is derived as

$$\text{SNR}_{CBF} = \frac{\rho_s \sigma^2 \sigma^2}{\sigma_\Sigma^2 \sigma_\Sigma^2 + \sigma_d^2},$$

where $\sigma_\Sigma$ is a singular value of $\Sigma$. On the other hand, when the channel vectors $g$ and $h^H$ are singular value decomposed as $u_1 \sigma g$ and $h^H$, respectively, without loss of generality, the received signal $y_d$ in (1) can be rewritten for the HBF as follows:

$$y_d = \alpha \sqrt{\rho_s} \alpha \sigma g v_1 \Psi^H w x + \alpha g z_r + z_d,$$

where $\sigma_h$ is an unique singular value for $h$. Assuming that SN and DN know the channel $h$ and $g$, respectively, SN and DN can obtain $v_{h1}$ and $u_1$ as an optimal beamforming and combining vector, respectively. Then, the combined signal of the HBF can be written as

$$r = \alpha \sqrt{\rho_s} \sigma_h x + \alpha u_{h1}^H g z_r + u_{h1}^H z_d.$$
and the average received SNR can be derived as

$$\text{SNR}_{HBF} = \frac{\rho_s \sigma^2_p}{\alpha^2 \sigma^2_p + \sigma^2_d}$$

(7)

Noting that $\sigma^2_p$ in (7) and $\sigma^2$ in (5) are identical, which can be easily shown, then $\text{SNR}_{CBF} = \text{SNR}_{HBF}$. In the following section, we compare the MSEs of the channel estimation for CBF and HBF systems after describing the channel estimation scenario for each system.

### IV. Channel Estimation Scenario and Performance Analysis Under Imperfect CSI

In practical wireless communications, a channel estimation procedure is necessary, and the performance degradation is inevitable due to channel uncertainty from estimation errors. This section focuses on the scenario and performance of channel estimation for the CBF and HBF systems.

Firstly, we consider the CBF system. As we mentioned in the previous section, the SN should know the effective channel $gh^H$ for beamforming in (4). The DN can easily estimate the downlink channel $gh^H$ by using a training sequence, termed $T_{CBF1}$ in Fig. 2. Since the up- and down-link channels of TDD systems are symmetrical, the effective channel $gh^H$ can be estimated at the SN by estimating $gh^H$ through the training sequence from the DN to the SN, termed $T_{HBF2}$. In order to estimate the effective channel, we assume that the SN and DN transmit training sequence $s(k) \in \mathbb{C}^{N_s \times 1}$ during $L$ symbol periods, where the time index $0 \leq k \leq L - 1$. To estimate the independent elements of the effective channel matrix $gh^H$ or $gH$, the condition should be satisfied, $L \geq N_s$, which implies that at least as many as measurements as unknowns are needed. For the purpose of simple derivation, the received signal can be rewritten in matrix form as follows:

$$Y_d = \alpha \sqrt{\rho_s} gh^H S + \alpha gZ^T + Z_d = HS + Z,$$

(8)

where the transmitted signal matrix $Y_d = \{y_d(0) \cdots y_d(L-1)\}$; the transmitted training matrix $S = \{s(0) \cdots s(L-1)\}$; the noise vector $Z^T = \{z_d(0) \cdots z_d(L-1)\}$ at the RN; and the noise matrix $Z_d = \{z_d(0) \cdots z_d(L-1)\}$ at the DN. On the second equality in (8), we define the effective channel matrix $\alpha \sqrt{\rho_s} gh^H$ as $H$ and the noise matrix $\alpha gZ^T + Z_d$ as $Z$. Now, we evaluate the MSE of estimation for the effective channel. Assuming that $S$ is a full rank, the maximum-likelihood estimate for the effective channel matrix can be written as $\hat{H} = Y_d S^H$, where $S^H = (SS^H)^{-1}$, which is the pseudo inverse of $S$. Then, the MSE of channel estimation for the DN in a CBF system can be written as follows:

$$\text{MSE}_{CBF, DN} \triangleq \frac{1}{\eta^2} \mathbb{E} \left[ \| \hat{H} - H \|^2 \right]$$

$$= \frac{\alpha^2 \sigma^2_p \sigma^2_p + N_s \sigma^2_d}{\eta^2} \text{tr}\left\{ (S^H S)^{-1} \right\},$$

where the normalization term $\eta^2$ is given by $\alpha^2 \rho_s N_s N_d$. The obtained $\text{MSE}_{CBF, DN}$ is affected by the channel realization $gh^H$ and $\text{tr}\{(S^H S)^{-1}\}$. Here, we consider that a training sequence matrix satisfies a power constraint and is constructed by an optimal form as follows [9]:

$$S^H S = \frac{L}{N_s} I_{N_s},$$

(10)

where $I_n$ represents an $n$-dimensional identity matrix. The condition (10) implies the training sequence matrix must have orthogonality, and we average out the MSE in (9) over channel realization $gh^H$. The MSE of CBF can be derived as

$$\text{MSE}_{CBF, DN} = \frac{N_s}{L \rho_s^2} (\alpha^2 \sigma^2_p + \sigma^2_d).$$

(11)

Using $\alpha$ in (2), $\text{MSE}_{CBF, DN}$ in (11) can be rewritten as follows:

$$\text{MSE}_{CBF, DN} = \frac{N_s \sigma^2_p}{L \rho_r} + \frac{N_s \sigma^2_p}{L \rho_r} \left(1 + \frac{\sigma^2_d}{\rho_r}\right)$$

$$= \frac{N_s}{L \gamma_{dr}} + \frac{N_s}{L \gamma_{rs}} \left(1 + \frac{1}{\gamma_{sr}}\right),$$

(12)

where we define the transmit-symbol-to-noise-power ratio as $\gamma_{ba}$ from node $a$ to $b$ and the characters $s$, $r$, and $d$ stand for source, relay, and destination, respectively. Similarly, we can derive the MSE of channel estimation for the SN in a CBF system as follows:

$$\text{MSE}_{CBF, SN} = \frac{N_s}{L \gamma_{sr}} + \frac{N_s}{L \gamma_{rd}} \left(1 + \frac{1}{\gamma_{sr}}\right).$$

(13)

Secondly, we consider the HBF system. As we mentioned previously, the SN and DN should know channels $h^H$ and $g$, respectively for the transmit beamforming in (6). Here, we introduce a pilot emitting relay which transmits the training sequences, such as pilots. By using the emitted pilot from RN, termed $T_{HBF}$ in Fig. 2, the SN and DN can independently estimate the channels $h^H$ and $g$, respectively. Thus, channel estimation performances at the SN and DN depend only on the corresponding SNRs of RN-SN and of RN-DN, respectively. Now, we derive the MSE of the channel estimation between the RN and DN. The RN emits L pilot symbols, in which implies that the same power of the training signal per time is used for the CBF and HBF systems. When pilot signals $p(k)$, which $E[p(k)]=0$ and $E[p(k)^2]=1$, are emitted (transmitted) during $L$ symbol periods from the RN, the received signal at the DN can be written as follows:

$$Y_d = \sqrt{\rho_s} gp^T + Z,$$

where $p^T = \{p(0) \cdots p(L-1)\}$ and the noise matrix $Z = \{z_d(0) \cdots z_d(L-1)\}$, in which $E[Z^H Z] = N_s \sigma^2_d I_L$. In the
same manner of channel estimation for CBF, the estimate of channel vector, \( \mathbf{g} \), can be obtained as \( Y_d (\mathbf{p} T) \). Then, the MSE of channel estimation for the DN in a HBF system can be derived as follows:

\[
\text{MSE}_{HBF,DN} = \frac{1}{\eta_2^2} \mathbb{E}[\|\mathbf{g} - \mathbf{g}\|^2] = \frac{1}{\eta_2^2} \mathbb{E}[\|\mathbf{Z} (\mathbf{p}^T) \|^2]
\]

\[
= \frac{1}{\eta_2^2} L \mathbf{p}^T \mathbb{E} [\mathbf{Z}^H \mathbf{Z}] \mathbf{p} = \frac{\sigma_d^2}{L \gamma_{dr}} = \frac{1}{L \gamma_{dr}},
\]

where the normalization term \( \eta_2^2 \) is given by \( \rho_s N_d \). Similarly, we can derive the MSE of channel estimation for the SN in a HBF system as

\[
\text{MSE}_{HBF,SN} = \frac{1}{\eta_2^2} \mathbb{E}[\|\mathbf{h} - \mathbf{h}\|^2] = \frac{1}{L \gamma_{sr}}.
\]

Finally, we compare the MSEs of channel estimation for CBF and HBF. In the case of CBF, training signals can be easily corrupted by additive noises at the RN since training signals should go through RN (see the second term of the second equality in (12) and the equality in (13)). However, since training signals emitted from RN do not include the RN’s additive noise, channel can be estimated accurately compared to that of CBF. To compare MSEs more precisely, we derive the difference of MSEs of CBF and HBF as

\[
\text{MSE}_{CBF,DN} - \text{MSE}_{HBF,DN} = \frac{N_s - 1}{N_s L \gamma_{dr}} + \frac{1}{L \gamma_{rs}} (1 + \frac{1}{\gamma_{dr}}) > 0
\]

from (12) and (14). Also, we can easily derive that \( \text{MSE}_{CBF,SN} > \text{MSE}_{HBF,SN} \), from (13) and (15). Consequently, it can be surmised that the system performance of HBF is better than that of CBF in a practical channel estimation scenario. Fig. 3 shows the analytic and experimental results of MSE for DN in CBF and HBF systems when \( N_s = L = 8 \). From this figure, it can be seen that the MSE of CBF decreases as \( \gamma_{rd} \) increases until a certain point; however, MSE of HBF decreases linearly as \( \gamma_{rd} \) increases. Additionally, Fig. 3 shows that our analysis and simulation results match well.

V. PERFORMANCE EVALUATION AND DISCUSSION

In this section, we show the BER performance of the CBF and HBF systems over transmit-symbol-to-noise power ratios \( \gamma_{dr} \) and \( \gamma_{rs} \). In simulation, we consider a beamforming system with a amplify-and-forward relay, as shown in Fig. 1. We also examine BER with various numbers of SN antennas, \( N_s \), when the number of DN antennas is two. The SN performs beamforming with \( N_s \) transmit antennas and the DN recovers the transmitted data signal, which is modulated by quadrature phase shift keying, with two receive antennas.

Fig. 4 shows the BER over \( \gamma_{dr} \) under the perfect CSI. The BERs of CBF and HBF are identical as we analyzed in (5) and (7). It is clear that the BER is a monotonic decreasing function of \( \gamma_{dr} \); however, error floors are observed in the region where \( \gamma_{dr} \) is approximately greater than \( \gamma_{rs} \). From this observation of error floors, it can be seen that the BER performance is limited by the weaker link.

Now, we examine the BER with a CSI estimation. Maximum-likelihood channel estimation, which was described in Section IV is used in a simulation. The length of the pilot sequence is \( N_s \), i.e., \( L = N_s \), which is the minimum number to estimate multiple channels, as we mentioned in Section IV. For fair comparison, the same length of pilot signals is used for the CBF and HBF systems, as shown in Fig. 2. Here, we assumed that the DN knows the effective channel gain, which can be estimated by the additional training sequence weighted by the beamforming vector. Actually, due to the channel uncertainty, the effective channel gain is not a real value; thus, it should be estimated for equalizing the post-processed signal at the DN.

In Fig. 5, the BER curves for CBF and HBF are plotted versus \( \gamma_{dr} \) with fixed \( \gamma_{rs} \) by 10 dB. Here, the benefit of the proposed CBF compared to the conventional CBF can be seen. With the same number of transmit antennas, \( N_s \), the HBF can achieve lower BER than CBF over any \( \gamma_{dr} \) region. Moreover, the BER performance gap between the CBF and HBF increases as the number of transmit antennas increases. This performance gap comes from the difference of channel estimation performance, which is mainly affected by \( \gamma_{rs} \).
the number of transmit antennas $N_s$, as analyzed in Section IV. Fig. 6 shows the BER performance over various $\gamma_{rs}$ with fixed $\gamma_{dr}$ by 20 dB. The BER performance gap between the CBF and HBF increases as $\gamma_{rs}$ increases by certain point, which is determined by the error floor caused by the limitation of $\gamma_{dr}$. However, it is clearly observed that the BER performance gains of HBF over CBF are obtained for the entire $\gamma_{rs}$ range. Additionally, we compare the CBF and HBF systems employing RN and CBF without RN in Fig. 7 under the cellular environment shown in Table I. As we expected, the proposed CBF has the best BER performance when the channel is estimated.

Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell radius</td>
<td>1 Km</td>
</tr>
<tr>
<td>RN position from SN</td>
<td>0.5 Km</td>
</tr>
<tr>
<td>SN and RN Tx power</td>
<td>23 dB</td>
</tr>
<tr>
<td>Shadowing STD (NLOS/LOS)</td>
<td>3/8.9 dB</td>
</tr>
<tr>
<td>Rx noise level</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>Propagation Model</td>
<td>SN-DN: 128.1+37.6log_10(d)</td>
</tr>
<tr>
<td>(path-loss, in dB)</td>
<td>SN-RN: 128.1+28.8log_10(d)</td>
</tr>
<tr>
<td>(d in kilometers)</td>
<td></td>
</tr>
</tbody>
</table>

VI. CONCLUSION

A pilot emitting amplify-and-forward relay is introduced, and the HBF method is proposed to employ a relay into the conventional beamforming systems. It was analytically and experimentally shown that the performance of the proposed HBF method is identical to that of the conventional CBF when the CSI is perfect on both of the SN and DN. Furthermore, we showed that the CSI estimation performance of the proposed HBF method using pilots emitted from the relay is better than that of the conventional CBF method using pilots from the SN and DN. Consequently, the proposed HBF method obtains better BER performance than CBF in the practical CSI estimation scenario.

REFERENCES