Multiuser Scheduling Based on Reduced Feedback Information in Cooperative Communications

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Abstract— We introduce a multiuser (MU) scheduling method for amplify-and-forward (AF) relay systems, which opportunistically selects both the transmission mode, i.e., either one- or two-hop transmission, and the desired user. A closed-form expression for the average achievable rates is derived under two transmissions with MU scheduling, and its asymptotic solution is also analyzed in the limit of large number \( N \) of mobile stations (MSs). Based on the analysis, we perform our two-step scheduling algorithm: the transmission mode selection followed by the user selection that needs partial feedback for instantaneous signal-to-noise ratios (SNRs) to the base station (BS). We also analyze the average SNR condition where the MU diversity gain is fully exploited. In addition, it is examined how to further reduce a quantity of feedback under certain conditions. The proposed scheduling algorithm shows the comparable achievable rates to those of optimal one using full feedback information, while its required feedback information is reduced below half of the optimal one.

I. INTRODUCTION

Relayed transmission techniques have the advantages of enhancing the end-to-end link quality in terms of capacity and extending the coverage [1]. One of the simplest relay protocol is two-hop relaying which assists the communication between a base-station (BS) and a mobile station (MS) [2]. There are lots of prior works to consider a two-hop relay cooperation, which include a variety of novel techniques such as distributed space-time coding [3], [4] and relay station (RS) selection [5], where one BS, one MS, and many RSs are deployed in the system. The disadvantage of two-hop relaying is that resources, e.g., time and frequency, are wasted twice compared to those of one-hop transmission. The two-hop relay scheme thus induces the pre-log factor \( \frac{1}{2} \) [6] with respect to the achievable rates, and cannot always guarantee a better system throughput compared to that of one-hop transmission. Hence, a proper scheduling, which selects the transmission mode, i.e., either one- or two-hop transmission, can be adopted to maximize the system throughput.

To improve the throughput even further, the multiuser (MU) diversity gain can be utilized by the randomness of fading since there are a large number of MSs in MU environments of cellular systems: opportunistic scheduling [7], opportunistic beamforming [8], and random beamforming [9] in broadcast channels. It has also been studied in [10] when a two-hop decode-and-forward (DF) relay system is considered and the BS-RS link is assumed to have the additive white Gaussian noise (AWGN). Under MU systems with RS support, the optimal strategy is that the BS simultaneously selects both the transmission mode and the desired MS among all users, based on instantaneous signal-to-noise ratios (SNRs) of one- and two-hop links, thereby requiring quite a lot of feedback information.

In this paper, we propose an MU scheduling method based on efficiently reduced feedback information for two-hop amplify-and-forward (AF) relay systems. First, the transmission mode is selected such that we have higher average achievable rates between two transmissions with MU scheduling. Next, the desired MS is selected opportunistically, based on partial feedback information to the BS, which includes instantaneous SNRs of either all of the one- or two-hop links. To construct a scheduling criterion, we derive a closed-form expression for the average achievable rates under two transmission schemes with MU scheduling. The asymptotic average achievable rates are also analyzed in the limit of large \( N \). From our analysis, we have the following interesting results: as \( N \) increases, the average achievable rates for two-hop transmission are either upper-bounded by a constant or unbounded because of MU diversity gain—the link condition where the MU diversity gain is fully exploited is given by a function of the average SNRs and \( N \). Based on the asymptotic results, it is also examined how to further reduce a quantity of feedback information under certain conditions. Then, a computer simulation is evaluated to verify the performance of our scheduling. We may conclude that the proposed scheduling scheme shows the comparable achievable rates to those of optimal one using full feedback information, while its required feedback information is fairly reduced below half of the optimal one.

The organization of this paper is as follows. Section II describes the system and channel model. In Section III, the proposed MU scheduling algorithm is shown. The average achievable rates under our MU scheduling is analyzed and a modified scheduling method is also shown in Section IV. Section V presents computer simulation results. Finally, Section VI summarizes this paper with some concluding remarks.

II. SYSTEM AND CHANNEL MODEL

Fig. 1 shows the MU downlink system which consists of one BS, one RS, and \( N \) MSs in cellular environments. For one-hop transmission, we perform a direct transmission from the BS to a certain MS. Then, the received signal at the \( n \)-th
MS is given by
\[ y_{1,n} = h_{bm,n} \sqrt{P_b} x + w_{m,n}, \quad n = 1, \cdots, N, \]  
where \( h_{bm,n} \) is the complex channel between the BS and the \( n \)-th MS, \( P_b \) is the transmit power at the BS, \( x \) is the transmit signal, and \( w_{m,n} \) denotes the complex AWGN at the \( n \)-th MS, which is distributed as \( \mathcal{CN} \left( 0, \sigma_b^2 \right) \). For two-hop transmission, the communication is performed from the BS to one MS through the RS. We assume the RS operates in half-duplex mode, i.e., may not receive and transmit simultaneously at the same time. Then, at the first time slot, the BS transmits its data to the RS, and at second time slot, the RS amplifies and forwards the data to the MS. For simplicity, we do not consider the direct-path from the BS to MSs for two-hop transmission. The received signal at the \( n \)-th MS is then given by
\[ y_{2,n} = h_{rm,n} g \left( h_{br} \sqrt{P_b} x + w_r \right) + w_{m,n}, \quad n = 1, \cdots, N, \]  
where \( h_{rm,n} \) and \( h_{br} \) are the complex channels between the RS and the \( n \)-th MS, and the BS and the RS, respectively, \( g \) is the amplification factor at the RS, and \( w_r \) denotes the complex AWGN at the RS, distributed as \( \mathcal{CN} \left( 0, \sigma_r^2 \right) \). The channel gains \( h_{bm,n} \) and \( h_{rm,n} \) for \( n = 1, \cdots, N \) and \( h_{br} \) are independent and identically distributed (i.i.d.) and frequency-flat fading where all the distribution is assumed to be \( \mathcal{CN} \left( 0, 1 \right) \). In our model, the amplification factor \( g \) at the RS is represented as [2]
\[ g = \frac{\sqrt{P_r}}{\sqrt{h_{br}^2 P_b + \sigma_r^2}}, \]  
where \( P_r \) is the transmit power at the RS. Suppose that all the average SNRs (based on geographic location) and the total number \( N \) of MSs are available at the BS in advance and an instantaneous SNR including channel gains is only acquired via feedback. In addition, we assume that multiple MSs are selected in the MU system such that both the average BS-MS and RS-MS SNRs are the same for all MSs, i.e., \( \bar{\gamma}_{bm} = \frac{P_b}{\sigma_{b,m,1}^2} = \cdots = \frac{P_b}{\sigma_{b,m,N}^2} \) and \( \bar{\gamma}_{rm} = \frac{P_r}{\sigma_{r,m,1}^2} = \cdots = \frac{P_r}{\sigma_{r,m,N}^2} \). It means that all the selected users are located on the place having the same radius from both the BS and the RS. For simplicity, we do not perform any power control over time.\(^1\)

### III. PROPOSED MU SCHEDULING

In this section, we propose the MU scheduling method that efficiently reduces feedback information of instantaneous SNRs from MSs to the BS. The BS decides both the transmission mode and the desired MS based on a scheduling criterion. Let \( C_i \) denote the average achievable rates for \( i \)-hop transmission \((i = 1, 2)\) when an MS is selected such that it has the maximum instantaneous SNR among \( N \) MSs. Then, \( C_i \) is expressed as a function of the average SNRs \((\bar{\gamma}_{bm}, \bar{\gamma}_{rm}, \bar{\gamma}_{br}\) and \( \bar{\gamma}_{br} = \frac{P_r}{\sigma_r^2} \)) and \( N \), which will be analyzed in Section IV. Our scheme is then composed of two-steps, and its procedure is as follows:

#### Step 1. Transmission mode selection

The transmission mode \( i \) is given by
\[ \hat{i} = \arg \max_{i \in \{1, 2\}} C_i. \]  
Note that the decision is based on a lookup table depending on parameters \( \bar{\gamma}_{bm}, \bar{\gamma}_{rm}, \bar{\gamma}_{br} \), and \( N \) (see TABLE I). Specifically, \( C_i \) for each hop is numerically computed by putting these parameters, and \( \hat{i} \) having a higher rate is then selected. For example, when \( N = 15 \), \( \bar{\gamma}_{bm} = 0 \) dB, \( \bar{\gamma}_{rm} = 20 \) dB, and \( \bar{\gamma}_{br} = 30 \) dB, \( \hat{i} \) becomes 2 (two-hop).

#### Step 2. User selection

The BS requests the instantaneous SNRs of the corresponding link to all the MSs. For one-hop transmission, the instantaneous BS-MS SNRs \( \tilde{\gamma}_{bm} | h_{bm,n} |^2 \) for \( n = 1, \cdots, N \) should be fed back to the BS. For two-hop transmission, the BS only needs the instantaneous BS-RS-MS SNRs, given by
\[ \frac{\bar{\gamma}_{br} | h_{br} |^2 | \tilde{\gamma}_{rm} | h_{rm,n} |^2}{\bar{\gamma}_{br} | h_{br} |^2 + \bar{\gamma}_{rm} | h_{rm,n} |^2 + 1}. \]  
Based on feedback information, the BS selects one MS that has the maximum instantaneous SNR of the corresponding link.

\(^1\)In [11], it is shown that the temporal power control gives the marginal gain for the average achievable rates under fading environments compared to the fixed power case at high SNRs. In our system, since an MU scheduling is performed and thus the user having a high SNR is selected, the power control gain will be negligible when \( N \) becomes sufficiently large.
For comparison, the optimal scheduling, based on full feedback of the instantaneous SNRs, is considered. In this case, the transmission mode and the desired user are selected simultaneously in a sense of maximizing the instantaneous achievable rates for a given channel realization. In Section V, it will be shown that the proposed scheduling algorithm always has much higher average achievable rates than those of either one- or two-hop transmission, which is rather obvious, while both have the same amount of feedback, and has comparable performance to that of the optimal one that requires a twofold amount of feedback.

IV. ACHIEVABLE RATE ANALYSIS

The average achievable rates of both one- and two-hop transmissions with MU scheduling are analyzed. A closed-form expression for the average achievable rates is first derived, and its asymptotic behavior is also shown in the limit of large $N$. In addition, based on the asymptotic result, we show another MU scheduling scheme that can further reduce feedback information under certain cases.

A. One-hop Transmission

From (1), the average achievable rates $C_1$ of one-hop transmission are given by

$$ C_1 = \mathbb{E} \left[ \log_2 \left( 1 + \text{SNR}_{1,\text{max}} \right) \right], $$

where $\text{SNR}_{1,\text{max}} = \tilde{\gamma}_{bm} \mathcal{h}_{bm,n}^2$ for $n = 1, \ldots, N$. In the following lemma, we derive a closed-form expression of (6).

**Lemma 1:** Suppose we perform the one-hop transmission with MU scheduling. Then, the average achievable rates $C_1$ are written as

$$ C_1 = \frac{N}{\ln 2} \sum_{n=0}^{N-1} (-1)^n \left( \frac{N-1}{n} \right) \frac{\gamma_{bm}}{n+1} E_1 \left( \frac{n+1}{\gamma_{bm}} \right), $$

where $\gamma_{bm} = \frac{P_b}{\sigma_b^2}$ and $E_1(x) = \int_{x}^{\infty} e^{-t} dt$ is the exponential integral function.

**Proof:** The proof essentially follows that in [12].

Note that $C_1$ is given by the function of the average BS-MS SNR $\tilde{\gamma}_{bm}$ and the number $N$ of MSs. For large $N$, the average achievable rates $C_1$ in (7) are asymptotically given by

$$ \log_2 \left( 1 + \tilde{\gamma}_{bm} \ln N \right) $$

with high probability [8]. In this case, the MU diversity gain can be fully exploited for any average SNRs, i.e., link conditions.

B. Two-hop Transmission

We first show the maximum instantaneous SNR, termed $\text{SNR}_{2,\text{max}}$, for two-hop transmission with MU scheduling. Using (2), we obtain $\text{SNR}_{2,\text{max}}$ as follows:

$$ \text{SNR}_{2,\text{max}} = \max_{n=1,\ldots,N} \frac{\gamma_{br} \gamma_{rm,n}}{\gamma_{br} + \gamma_{rm,n} + 1}, $$

where $\gamma_{br} = \frac{P_b}{\sigma_r^2}$ and $\gamma_{rm,n} = \tilde{\gamma}_{rm} \mathcal{h}_{rm,n}^2$. The main characteristics for the right-hand-side (RHS) of (9) are shown in the following lemma.

**Lemma 2:** The function

$$ f(\gamma_{rm,n}) = \frac{\gamma_{br} \gamma_{rm,n}}{\gamma_{br} + \gamma_{rm,n} + 1} $$

is monotonically increasing with respect to $\gamma_{rm,n}$.

**Proof:** The result is simply given by taking the first derivative of $f(\gamma_{rm,n})$ with respect to $\gamma_{rm,n}$.

Thus, the average achievable rates $C_2$ of two-hop transmission are given by

$$ C_2 = \mathbb{E} \left[ \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{2,\text{max}} \right) \right] $$

$$ = \mathbb{E} \left[ \frac{1}{2} \log_2 \left( 1 + \frac{\gamma_{br} \max_{n=1,\ldots,N} \gamma_{rm,n}}{\gamma_{br} + \max_{n=1,\ldots,N} \gamma_{rm,n} + 1} \right) \right], $$

where the second equality comes from Lemma 2, and the exact closed-form expression of (11) is derived in the following proposition.

**Proposition 1:** Suppose we perform the two-hop transmission with MU scheduling. Then, the average achievable rates $C_2$ are written as

$$ C_2 = \frac{N}{2 \ln 2} \sum_{n=0}^{N-1} (-1)^n \left( \frac{N-1}{n} \right) \times $$

$$ \left\{ \frac{1}{\gamma_{br}} \mathcal{E}_1 \left( \frac{1}{\gamma_{br}} \right) n \mathcal{E}_1 \left( \frac{n+1}{\gamma_{br}} \right) \right\}, $$

where $\frac{1}{\gamma_{br}} = \frac{P_b}{\sigma_r^2}$ and $\gamma_{rm} = \frac{P_r}{\sigma_r^2}$.

**Proof:** (Brief sketch) The probability density function (pdf) $p_{\gamma_{br}}(x)$ of the random variable $\gamma_{br}(= x)$ is exponentially distributed and is given by

$$ p_{\gamma_{br}}(x) = \frac{1}{\gamma_{br}} e^{-\frac{x}{\gamma_{br}}}, $$

and the pdf $p_{\gamma_{rm,\text{max}}}(y)$ of $\gamma_{rm,\text{max}} = \max_n \gamma_{rm,n}(= y)$ is given by

$$ p_{\gamma_{rm,\text{max}}}(y) = \frac{N}{\gamma_{rm}} e^{-\frac{y}{\gamma_{rm}}} \left( 1 - e^{-\frac{y}{\gamma_{rm}}} \right)^{N-1} $$

$$ = \frac{N}{\gamma_{rm}} \sum_{n=0}^{N-1} (-1)^n \left( \frac{N-1}{n} \right) e^{-\frac{(n+1)y}{\gamma_{rm}}}, $$

where the first and second equalities hold due to the order statistics [13] of an exponential random variable and the binomial theorem, respectively. Using (13) and (14) and then taking the integrals of the logarithmic term in (11) with respect to $x$ and $y$, we finally obtain the result in (12).

In addition, we examine the asymptotic behavior of the average achievable rates $C_2$ in (12) for large $N$. Unlike the asymptotic result for one-hop transmission, it is shown that the full MU diversity gain is not always guaranteed for two-hop case. The following proposition shows the link condition where the MU diversity gain is fully exploited in an asymptotic manner.
Proposition 2: Suppose we perform the two-hop transmission with MU scheduling. When the number $N$ of MSs becomes large and the average RS-MS SNR $\bar{\gamma}_{br}$ does not scale with $N$, the average achievable rates $C_2$ shown in (12) are asymptotically given by

$$C_2 \approx \begin{cases} c^{\frac{1}{\gamma_{br}} - E_{1}\left(\frac{1}{\gamma_{br}}\right)} & \text{if } \bar{\gamma}_{br} = o(\ln N) \\ \frac{1}{2} \log_2 \left(1 + \beta \bar{\gamma}_{rm} \ln N\right) & \text{if } \ln N = O(\bar{\gamma}_{br}) \end{cases}$$

with high probability, where $\bar{\gamma}_{br} = \frac{P_m}{\sigma^2}$ and $\bar{\gamma}_{rm} = \frac{P_m}{\sigma^2}$. Here, $\beta = 1$ if $\ln N = o(\bar{\gamma}_{br})$, and $0 < \beta < 1$ if $\ln N = \mathcal{O}(\bar{\gamma}_{br})$ for some constant $C > 0$.

Proof: (Brief sketch) When $N$ is sufficiently large, the maximum $\max_n |h_{rm,n}|^2$ of an exponential random variable scales as $\ln N$ with high probability [8]. By applying the result to (11), we get

$$C_2 \approx \mathbb{E} \left[ \frac{1}{2} \log_2 \left(1 + \text{SNR}'_{2,\text{max}}\right) \right],$$

(16)

where $\text{SNR}'_{2,\text{max}} = \frac{\bar{\gamma}_{br} |h_{br}|^2 \gamma_{rm}}{\ln N}$. We first consider the case where $\bar{\gamma}_{br} = o(\ln N)$. Then, we get

$$\text{SNR}'_{2,\text{max}} \approx \frac{\bar{\gamma}_{br} |h_{br}|^2 \gamma_{rm}}{\ln N},$$

(17)

where the approximation holds since the random variables $\frac{|h_{br}|^2}{\ln N}$ and $\frac{1}{\ln N}$ tend to zero with high probability under the condition. Thus, (16) is rewritten as

$$C_2 \approx \mathbb{E} \left[ \frac{1}{2} \log_2 \left(1 + \bar{\gamma}_{br} |h_{br}|^2\right) \right] = \frac{1}{2} \int_0^\infty \log_2 \left(1 + x\right) \varphi_{h_{br}}(x) \, dx.$$

(18)

Plugging (13) (shown in the proof of Proposition 1) into (18) and taking the integral with respect to $x$, we simply get the first equation of (15). When $\ln N = O(\bar{\gamma}_{br})$, following the approach similar to the first case, we obtain

$$\text{SNR}'_{2,\text{max}} \approx \begin{cases} \bar{\gamma}_{br} \ln N, & \text{if } \ln N = o(\bar{\gamma}_{br}) \\ \beta \bar{\gamma}_{rm} \ln N, & \text{if } \ln N = C\bar{\gamma}_{br}, \end{cases}$$

(19)

where $\beta = \frac{|h_{br}|^2}{|h_{br}|^2 + C}$ and $0 < \beta \leq 1$. Here, if $\ln N = o(\bar{\gamma}_{br})$, then $C' = 0$, otherwise $C' > 0$. This results in the second equation of (15), which completes the proof.

If the average BS-RS SNR $\bar{\gamma}_{br}$ is relatively much smaller than $\ln N$, i.e., $\bar{\gamma}_{br} \ll \ln N$, then the MU diversity gain is not fully exploited. It means that increasing the number $N$ of MSs beyond a certain value is not beneficial in the system performance under the condition. Especially, when $\bar{\gamma}_{br}$ is fixed (and thus does not scale with $N$), $C_2$ is bounded by a constant even for large $N$. Hence, we may conclude that it is not desirable for all MSs to feedback the instantaneous SNRs of the BS-RS-MS link to the BS. On the other hand, if $\bar{\gamma}_{br}$ scales relatively faster than $\ln N$, i.e., $\bar{\gamma}_{br} \gg \ln N$, then we can fully obtain the MU diversity gain as shown in the second equation of (15). In this case, a greater number of MSs reporting their instantaneous SNRs yields higher average achievable rates.

C. Modified MU Scheduling

Based on the asymptotic results, we introduce another MU scheduling further reducing feedback information of instantaneous SNRs under certain link conditions. Step 1 is the same as that of the originally proposed scheme while Step 2 is slightly modified as follows:

Step 2. User selection

When the transmission mode $\hat{i}$ is given by 1, the BS requests the instantaneous BS-MS SNRs to all the MSs. For $i = 2$, the scheduling strategy depends on the following link conditions.

- If $\bar{\gamma}_{br} = o(\ln N)$, $m \in \{1, \cdots, N\}$ randomly selected MSs feed back their instantaneous BS-RS-MS SNRs shown in (5) to the BS where $m$ is arbitrarily chosen depending on system parameters.
- Otherwise, the BS needs the instantaneous SNRs of all the MSs.

Since the MU diversity gain is not fully exploited under the condition $\bar{\gamma}_{br} = o(\ln N)$, full feedback of the instantaneous SNRs may not be much beneficial. In Section V, it will be shown that the modified scheduling method with sufficiently small $m$ has the almost same average achievable rates as those of the originally proposed one.

V. SIMULATION RESULTS

In this section, we perform a computer simulation to show the average achievable rates for some transmission strategies under consideration and then to demonstrate the advantage of our scheduling method.

Fig. 2 shows the average achievable rates versus the number $N$ of MSs when we perform either one- or two-hop transmission with MU scheduling. The numerical results have been examined for various SNRs: the average BS-RS SNR $\bar{\gamma}_{br}$ is 30 dB; the average BS-MS SNRs $\bar{\gamma}_{rm}$ are 0, 10, and 20 dB; and the average RS-MS SNRs $\bar{\gamma}_{br}$ are 10, 20, and 30 dB. We remark that the result for our proposed MU scheduling follows the outermost boundary of two curves for either one- or two-hop transmission with MU scheduling. When $\bar{\gamma}_{br} = 10$ dB, $\bar{\gamma}_{rm} = 30$ dB, and $N = 1$, the average achievable rates for two-hop transmission are higher than those for one-hop transmission. However, as $N$ increases, the result for one-hop case becomes higher due to more MU diversity gain. The analytical results, shown in (7) and (12) for one- and two-hop transmissions, respectively, are also shown in this figure. Their asymptotic behaviors are examined as follows: we get 4.5 from the first equation in (15); and the curve for $\bar{\gamma}_{rm} = 10$ dB is obtained from the second equation in (15) ($\beta = 1$ is assumed). It is seen that the numerical and analytical results match well for any average SNRs and $N$. Interestingly, when
The proposed scheme shows the same performance as that of the optimal one. However, it is easily seen that the proposed scheme always outperforms either one- or two-hop transmission.

\[ N \text{ increases over } 10 \text{ for } \gamma_{rm} = 30 \text{ dB, the numerical result is asymptotically upper-bounded by } 4.5. \text{ In this case, we may conclude that feedback from randomly chosen } 10 (= m) \text{ MSs provides the nearly same performance as the full feedback case from all the MSs. It is also seen that the analytical result in the second equation of (15) and the numerical one for two-hop transmission with } \gamma_{rm} = 10 \text{ dB match well for } N \geq 5. \]

Moreover, to verify the performance of our MU scheduling, the comparison for the average achievable rates between the optimal and proposed schedulings is evaluated in Fig. 3. It is assumed that \( \gamma_{br} = 30 \text{ dB, } \gamma_{rm} = 30 \text{ dB, and } \gamma_{brm} = 10 \text{ dB. } \) The optimal scheduling shows slightly better performance than that of the proposed one, which is given by the outermost boundary of two curves for either one- or two-hop transmission, especially on the crossover where two curves meet.\(^3\) However, it is easily seen that the proposed scheme always outperforms either one- or two-hop transmission.

\[^3\text{If there is no crossover between two curves under a certain link condition, the proposed scheme shows the same performance as that of the optimal one (which is not shown in this paper).}\]

VI. CONCLUSION

The MU scheduling method, opportunistically selecting both the transmission mode and the desired MS, has been proposed for two-hop AF relay systems. The scheduling criterion, based on efficiently reduced feedback information, was constructed by showing the closed-form expressions for the average achievable rates and their asymptotic solutions for large \( N. \) Under the scheduling, we analyzed the link condition where the MU diversity gain is fully exploited. The modified scheduling was also shown to further reduce a quantity of feedback under certain conditions. Finally, it was examined that the proposed algorithm has the nearly same achievable rates as those of optimal one, while its required feedback is fairly reduced below half of optimal one.

Further work in this area includes extending our MU scheduling method to multi-carrier systems, e.g., orthogonal frequency division multiplexing (OFDM).

REFERENCES