On the Use of Interpolated Second-Order Polynomials for Efficient Filter Design in Programmable Downconversion

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Abstract—Interpolated second-order polynomials (ISOP’s) are proposed to design efficient cascaded integrator-comb (CIC)-based decimation filters for a programmable downconverter. It is shown that some simple ISOP’s can effectively reduce the passband droop caused by CIC filtering with little degradation in aliasing attenuation. In addition, ISOP’s are shown to be useful for simplifying halfband filters that usually follow CIC filtering. As a result, a modified halfband filter (MHBF) is introduced which is simpler than conventional halfband filters. The proposed decimation filter for a programmable downconverter is a cascade of a CIC filter, an ISOP, MHBF’s, and a programmable finite impulse response filter. A procedure for designing the decimation filter is developed. In particular, an optimization technique that simultaneously designs the ISOP and programmable FIR filters is presented. Design examples demonstrate that the proposed method leads to more efficient programmable downconverters than existing ones.

Index Terms—Decimation filter, interpolated second-order polynomial (ISOP), modified halfband filter (MHBF), programmable downconverter, software radio.

I. INTRODUCTION

In contrast to most wireless communication systems which employ digital signal processing (DSP) only at baseband, software radio systems usually start DSP at an intermediate frequency (IF) band. By using programmable DSP chips at intermediate frequency as well as at baseband, software radio systems are flexible, and can efficiently support multiband and multistandard communications [1]–[3]. The input to an intermediate frequency stage of a software radio receiver is, in general, a wide-band signal, which is converted into a digital signal by bandpass sampling. The purpose of DSP at this stage is to isolate the signal of interest, which is usually a narrow-band signal, from a wide-band input, and to translate the signal down to baseband. For example, in the software radio receiver illustrated in Fig. 1, the analog input to the intermediate frequency stage is a wide-band signal with a bandwidth (BW) of 15 MHz and center frequency $f_c = 37.5$ MHz. After 30 Msamples/s (sps) bandpass sampling, the center frequency of the digital signal corresponds to 7.5 MHz.

This signal is passed through a programmable downconverter consisting of a digital mixer cascaded with a decimation filter. The programmable downconverter translates the signal down to baseband, isolates a narrow-band signal centered at 0 Hz (dc), and decimates it to lower the output sampling rate.

In software radios, the design of an efficient decimation filter is important because the input sampling rate of the filter is usually very high, and its passband and transition bandwidths are extremely narrow. For example, again referring to Fig. 1, if the signal of interest has a 30 kHz passband and the sampling rate is 50 Msps, then the passband width of the decimation filter is $0.6 \times 10^{-3}$ in normalized frequency. A popular approach to efficient decimation filter design is based on the use of the cascaded integrator-comb (CIC) decimation filter proposed by Hogenauer [4]. A programmable CIC filter is simple to implement, and can effectively reduce the aliasing effect caused by decimation. As pointed out in [5], however, this filter introduces a droop in the passband of interest. To overcome these difficulties, CIC filters are usually cascaded with a second decimating low-pass filter stage. Programmable FIR filters are used for this stage in [6] and [7]. To reduce the complexity of such programmable filters, fixed halfband decimation filters [8] are used in conjunction with a programmable FIR filter [9], [10]. In an attempt to avoid the use of a programmable filter at the second stage, K wentus et al. [5] replaced the CIC filter with a sharpened CIC filter that can significantly reduce the passband droop caused by CIC filtering. They employed only fixed coefficient halfband filters at the second stage. By using programmable sharpened CIC filters, this decimation filter can isolate input signals with different bandwidth, but its application is limited. For example, it is not applicable to multistandard communications in which decimation filters with different transition bandwidths are required. This is because the transition bandwidth provided by these halfband filters in [5] is fixed.

In this paper, a new CIC-based decimation filter is proposed as a useful alternative to the sharpened CIC filter in [5]. The proposed filter is a cascade of the CIC filter with the interpolated second-order polynomial (ISOP) filter. This ISOP filter, which was developed in [11] for efficient digital filter design, can significantly reduce the passband droop of the CIC filter. By employing a simple ISOP filter after CIC filtering, the filters at the second stage of the decimation filter—such as halfband filters and programmable FIR filters—can be considerably simplified. Some design examples will show...
Fig. 1. A software radio receiver employing a programmable downconverter.

Fig. 2. The CIC decimation filter: (a) the RRS filter \( H(z) \) is directly implemented and (b) the integrator and comb sections of the RRS are separated by the decimator.

that decimation filters with the ISOP filter can easily support multistandard communications, and are simpler to implement than existing ones.

The organization of this paper is as follows. In Section II, the ISOP filter is introduced, and its properties are discussed. In Section III, the method for designing decimation filters employing the CIC filter cascaded with the ISOP filter is developed. Section IV presents some design examples considering programmable downconverters for mobile communication.

II. CIC Decimation Filters Sharpened by ISOP’s

This section examines the design of ISOP filters following CIC filters after briefly reviewing CIC and sharpened CIC filters.

A. CIC and Sharpened CIC Filters

The CIC decimation filter [4]–[7] consists of cascaded recursive running sum (RRS) filters followed by a decimator, as shown in Fig. 2(a). The system function of the cascaded RRS filter is given by

\[
H(z) = \left( \frac{1-z^{MR}}{1-z^{-1}} \right)^L
\]  

(1)

where \( M \) is an integer decimation factor, and \( R \) is a differential delay and is a positive integer. The denominator and numerator terms of \( H(z) \) are referred to as the integrator and the comb filter, respectively. When implementing CIC filters, the integrator and comb sections are separated by the decimator, as shown in Fig. 2(b), to reduce computational load. The frequency response of \( H(z) \) is written as

\[
H(e^{j\omega}) = \left( \frac{1}{MR} \frac{1-e^{jMR\omega}}{1-e^{j\omega}} \right)^L
\]  

(2)

This response has nulls at multiples of \( f = 1/MR \), as shown in Fig. 3. These nulls provide natural attenuation of aliasing caused by the \( M \)-fold decimation since the frequency bands that are folded into the baseband by the decimation are centered around the nulls at multiples of \( f = 1/M \). The worst case aliasing occurs at the lower edge of the first aliasing band at \( f_A = 1/M - f_c \), where \( f_c \) is the passband width.

The sharpened CIC filter [5] is derived by replacing \( H(z) \) of the CIC filter in Fig. 2(a) with a sharpened filter \( H_s(z) = H^2(z)[3-2H(z)] \), which requires three CIC filters. In [5], only those CIC filters with an even \( L \) and \( R \) are considered. (The sharpening characteristic at passband is degraded if \( R \) is increasing, and an even \( L \) value is required to manage integer group delay.) This sharpening can significantly reduce passband droop and improve aliasing rejection, as can be seen in Fig. 3. The implementation of \( H_s(z) \) is, of course, considerably more expensive than that of \( H(z) \). In what follows, a simpler and more flexible sharpening technique than the sharpened CIC filter is introduced.

B. The CIC Filter Cascaded with the ISOP Filter

The system function of the ISOP filter \( P(z) \) is defined as

\[
P(z) = \frac{1}{|c+2|} \left( 1+cz^{-L} + z^{-2L} \right)
\]  

(3)
where \( I \) is a positive integer and \( c \) is a real number. \( P(z) \) is an interpolated version of the second-order polynomial

\[
S(z) = \frac{1}{|c+2|} (1 + cz^{-1} + z^{-2}),
\]

(4)

This polynomial has the following property, which is useful for filter sharpening.

**Property:** When \( c \) is real, the magnitude response of the polynomial \( S(z) \) is expressed as

\[
|S(e^{j\omega})| = \frac{1}{|c+2|} |c+2 \cos \omega|
\]

(5)

and is monotonically increasing in \( \omega \in [0, \pi] \) if \( c < -2 \). Due to the scaling factor \( 1/|c+2| \), the dc gain is always 1, and the slope of the magnitude response varies depending on the parameter \( c \).

The filter-sharpening characteristic of the ISOP filter stems from this property. The magnitude response of the ISOP filter is given by

\[
|P(e^{j\omega})| = \frac{1}{|c+2f_c|} |c+2f_c \cos \omega|.
\]

(6)

This is monotonically increasing in \( \omega \in [0, \pi/I] \) and is periodic with period \( 2\pi/I \), where \( I \) is an interpolation factor. The ISOP filter can compensate for the passband droop of the CIC filter, which is monotonically decreasing, in the frequency range \( \omega \in [0, \pi/I] \). To compensate for passband droop, the width of the monotonically increasing region \( \omega \in [0, \pi/I] \) should coincide with the input bandwidth \( 2\pi f_c \). This means that \( I = 1/(2f_c) \). In designing ISOP’s, it would be sufficient to consider only those \( I \) values satisfying

\[
1 \leq I \leq \left[ \frac{1}{2f_c} \right].
\]

(7)

If we set \( I = kM \), for \( k \) a positive integer, then the minima of the ISOP magnitude response occur at multiples of \( f = 1/kM \).

In this case, the location of every \( k \)th minimum coincides with those of the CIC Nulls at which aliasing bands are centered, and thus the aliasing rejection characteristic of the CIC decimation filter can be retained after ISOP filtering. When \( I = kM \), (7) becomes

\[
1 \leq k \leq \left[ \frac{1}{2Mf_c} \right]
\]

(8)

for a given \( M \). Fig. 4 illustrates \( |P(e^{j\omega})| \) for several values of \( k \) and \( c < -2 \). Note that the slope of \( |P(e^{j\omega})| \) increases as \( |k| \) is decreased and as \( k \) is increased. The maximum and minimum values of \( |P(e^{j\omega})| \), that can be obtained from (6), are \( (|k|+2)/(|k|-2) \) and 1, respectively.

Fig. 5 shows the cascade of CIC and ISOP filters. For this cascade, if the CIC filter is given, an optimal ISOP can be designed by using conventional filter design methods.
TABLE I
PASSBAND DROOP AND ALIasing ATTENUATION OF THE CASCADE, CIC, AND SHARPENED CIC FILTERS

<table>
<thead>
<tr>
<th>Filters with M = 8</th>
<th>$f_e = 1/8M$</th>
<th>$f_e = 1/4M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cascaded</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L = 4, R = 1$</td>
<td>0.0248</td>
<td>0.41</td>
</tr>
<tr>
<td>$L = 4, R = 2$</td>
<td>0.26</td>
<td>4.53</td>
</tr>
<tr>
<td>$L = 6, R = 1$</td>
<td>0.046</td>
<td>0.754</td>
</tr>
<tr>
<td>$L = 6, R = 2$</td>
<td>0.535</td>
<td>8.78</td>
</tr>
<tr>
<td>CIC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L = 4, R = 1$</td>
<td>0.88</td>
<td>3.59</td>
</tr>
<tr>
<td>$L = 4, R = 2$</td>
<td>3.64</td>
<td>15.64</td>
</tr>
<tr>
<td>$L = 6, R = 1$</td>
<td>1.33</td>
<td>5.39</td>
</tr>
<tr>
<td>$L = 6, R = 2$</td>
<td>5.45</td>
<td>23.45</td>
</tr>
<tr>
<td>Sharpend</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L = 2, R = 1$</td>
<td>0.062</td>
<td>0.84</td>
</tr>
<tr>
<td>CIC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L = 4, R = 1$</td>
<td>0.231</td>
<td>2.692</td>
</tr>
</tbody>
</table>

such as the modified Parks–McClellan method [12], [13] and linear programming [14], [15]. Specifically, for each integer $k$ satisfying (8), solve the following:

\[
\begin{align*}
\text{minimize} & \quad \delta \\
\text{subject to} & \quad |H(e^{j\omega}) \cdot P(e^{j\omega}) - 1| < \delta, \\
& \quad \text{for } 0 \leq \omega \leq 2\pi f_c
\end{align*}
\]

where $H(e^{j\omega})$ and $P(e^{j\omega})$ are the frequency responses of CIC and ISOP filters, respectively. Given $H(e^{j\omega})$, an optimal $P(e^{j\omega})$ minimizing $\delta$ can be obtained using well-known filter design tools. After solving (9) for each $k$, a $(k, c)$ pair associated with the smallest $\delta$ is chosen.

To examine the performance characteristics of the cascaded filter, this filter was designed for several values of $L$, $R$, and the input bandwidth $f_c$, and compared with CIC and sharpened CIC filters. The results are summarized in Table I. As $L$ and $R$ are increased, aliasing attenuation of these three filters is improved, but their passband droop is also increased. Both the cascaded and the sharpened CIC filters reduce the passband droop of CIC filtering at the expense of some degradation in aliasing rejection; between these two, the former can perform better than the latter. As an example, consider the cascaded filter with $L = 6$ and $R = 1$, and the sharpened CIC filter with $L = 2$ and $R = 1$. These filters employ the same number of RRS filters, and their computational complexities are almost identical. It is seen from Table I that the cascaded filter is better than the sharpened CIC filter both in reducing passband droop and in aliasing rejection. The cascade of CIC and ISOP filters, which has a very simple architecture, is a useful alternative to the sharpened CIC decimation filters.

C. ISOP Filters Sharpening Modified Halfband Filters

As mentioned in Section I, CIC decimation filters are usually followed by fixed halfband filters whose magnitude responses are symmetric with respect to $f = 0.25$. When the ISOP filter is employed, the symmetry requirement of the halfband filters can be relaxed by using the sharpening characteristic of ISOP filtering. For example, a low-pass filter with the following specification can be used in place of a halfband filter:

- passband: $f \in [0, f_p]$ \\
- stopband: $f \in [0.5 - f_p, 0.5]$ \\
- ripple: $\delta_1$ and $\delta_2$ for passband and stopband, respectively, $\delta_1 > \delta_2$
condition: magnitude response is monotonically decreasing in the passband. \( \text{(10)} \)

This low-pass filter, which will be referred to as the modified halfband filter (MHBF), has an asymmetric magnitude response as shown in Fig. 6. Since the magnitude response of the MHBF is monotonically decreasing in passband, the passband ripple \( \delta_2 \) becomes passband droop that can be reduced by ISOP filtering. MHBF with frequency response \( A(e^{j\omega}) \) is designed as follows:

\[
\begin{align*}
\text{minimize} & \quad \delta_1 \\
\text{subject to} & \quad |A(e^{j\omega})| < \delta_2 \quad \text{(in stopband)} \\
& \quad |A(e^{j\omega})| \text{ is monotonic} \quad \text{(in passband).} \end{align*}
\]

This problem can be solved by linear programming. When MHBF’s are employed after the cascade of CIC and ISOP filters, the ISOP should reduce the passband droop of the MHBF’s as well as that of the CIC filter. Such an ISOP can be designed as in (9). Details in designing ISOP filters are presented again in the following section, discussing the decimation filter design. It will be shown that the implementation of an MHBF can be considerably simpler than that of a halfband filter in spite of the fact that most coefficients of an MHBF are nonzero. (In conventional halfband filtering, about half of filter coefficients are zero.)

**III. OVERALL DECIMATION FILTER DESIGN**

An architecture of an overall decimation filter employing the cascade of CIC and ISOP filters is shown in Fig. 7. The filters following the ISOP filter consist of a multistage halfband decimation filter [8], a programmable FIR filter, and an interpolation filter. (This architecture is proposed in [9].) The multistage halfband decimation filter, shown in Fig. 8, is a cascade of decimation filters consisting of an MHBF followed by a 2-to-1 sample rate decimator. The MHBF’s in this block have fixed coefficients, and are reasonably simple to implement, especially in dedicated hardware because multiplierless implementation is possible by using techniques such as canonical signed digit (CSD) coefficients design [15], [16]. The programmable FIR filter provides flexibility for multistandard communication applications. Its implementation is often costly because it tends to have a long impulse response. Due to its programmability, multiplierless implementation is not recommended. Therefore, it is usually desirable to reduce the input rate of the programmable FIR filter as much as possible. The interpolation filter, which is sometimes optional, adjusts the output sampling rate as desired. The following presents some details for designing these filters.

**Multistage Halfband Decimation Filter Design:** Let the total number of available MHBF’s be \( J \). These filters are so ordered that \( f_{p1} < f_{p2} < \cdots < f_{pJ} \) where \( f_{pi} \) is the bandwidth of the \( i \)th MHBF. When designing the multistage decimator for a given application, \( m \) out of \( J \) MHBF stages are selected depending on the bandwidth \( Mf_c \) which is the bandwidth of the CIC filter output. To be specific, denote the index of the selected MHBF’s by \( s(i) \), \( 1 \leq i \leq m \) where \( s(i) \in \{1, 2, \ldots, J\} \). It is assumed that the bandwidth of \( s(1) < s(2) < \cdots < s(m) \). Then their bandwidth \( f_{p(i)} \) should meet

\[
f_{p(i)} > 2^{i-1}Mf_c, \quad \text{for all } 1 \leq i \leq m. \quad \text{(12)}
\]

The reason for this is follows: The first MHBF should pass the input signal with bandwidth \( Mf_c \). Thus, \( f_{p(1)} > Mf_c \). After 2-to-1 decimation, the bandwidth of the input to the second MHBF becomes \( 2Mf_c \), and thus the filter bandwidth \( f_{p(2)} \) should be larger than \( 2Mf_c \). The rest can be proved in the same manner. The decimation ratio provided by the multistage halfband decimation is \( 2^m \). An MHBF which is not selected but has bandwidth larger than \( f_{p(m)} \) can serve as a prefilter preceding the programmable FIR filter, after removing the 2-to-1 decimator. (The role of a prefilter is to reduce...
the computational burden of programmable FIR filtering.) For example, Fig. 8 shows MHBF1 and MHBF2 with their 2-to-1 decimators \( m = 2 \) and MHBF3 without its decimator as a prefilter.

**Determination of Decimation Factors** \( M \) and \( 2^m \): Given the desired decimation ratio, say \( D \), of the overall filter, it is necessary to determine \( m \) and \( M \) satisfying \( D = 2^m M \) (\( D < F_s / 2 f_c \)). The procedure for obtaining those values is as follows. First, the maximum possible value of \( m \), say \( m_{\text{max}} \), is determined by counting the number of MHBF’s satisfying the condition in (12). Then, for each \( m \in \{ m_{\text{0}} \leq m \leq m_{\text{max}} \} \) and the corresponding value of \( M \), the overall decimation filter is designed, and the pair \( (m, M) \) that leads to the filter with minimal complexity is chosen. When the desired decimation factor \( D \) is odd, one must set \( m \) to zero. In this case, one may consider a decimation factor \( 2^n D \) for \( n \) a small positive integer, instead of \( D \). This is possible since the interpolator following programmable FIR filtering can compensate for the additional \( 2^n \)-to-1 decimation.

**CIC Filter Design:** For a given decimation ratio \( M \), the differential delay \( R \) and the number of RRS stages \( L \) are so determined that the desired aliasing attenuation is met. Unlike the CIC filter design in [4], it is unnecessary to pay attention to passband droop of CIC filtering while deciding \( L \) and \( R \) because most passband droop can be reduced by ISOP filtering.

**Simultaneous Design of ISOP and Programmable FIR Filters:** After completing the design of the CIC and multistage halfband decimation filters, ISOP and programmable FIR filters can be simultaneously designed so that the overall decimation filter meets specifications. A procedure for designing these filters can be developed by extending the ISOP design problem in (9). Since the overall filter is conveniently specified with frequencies normalized by \( F_s \), which is the input rate of CIC filtering, the design problem is formulated with such normalized frequencies. Let \( G(e^{i\omega}) \) denote the frequency response of the cascaded CIC and multistage halfband decimation filters, and let \( H_d(e^{i\omega}) \) denote the desired frequency response of the overall decimation filter. In evaluating \( G(e^{i\omega}) \), decimation factors associated with it should be carefully considered. For example, when the number of selected MHBF stages is three \( m = 3 \), \( G(e^{i\omega}) \) is expressed as

\[
G(e^{i\omega}) = H(e^{i\omega}) A_{s(1)}(e^{iM\omega}) A_{s(2)}(e^{i2M\omega}) A_{s(3)}(e^{i4M\omega})
\]  

(13)

where the first term on the right is the frequency response of the CIC filter in (2), and \( A_{s(1)}(e^{i2^mM\omega}) \) is the frequency response of the \( i \)-th selected MHBF with aggregate decimation ratio \( 2^m M \). Considering decimation factors, the frequency response of programmable FIR filtering should be written in the form \( F(e^{i2^nM\omega}) \). The objective is to minimize the complexity of programmable FIR filtering under given filter specifications. Specifically, we consider the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \text{number of taps for programmable FIR filtering} \\
\text{subject to} & \quad |G(e^{i\omega})P(e^{i\omega})F(e^{i2^nM\omega}) - H_d(e^{i\omega})| < \delta_p \\
& \quad \text{in passbands}
\end{align*}
\]  

(14)

where \( \delta_p \) and \( \delta_s \) denote passband and stopband ripples, respectively; \( P(e^{i\omega}) \) is the frequency response of the ISOP filters in (6); and \( H_d(e^{i\omega}) \) is assumed to be zero in stopbands. The passband is given by \( f \in \{ 0, f_c \} \) where \( f_c \) is the signal bandwidth (see Fig. 3). The problem in (14) can be solved by linear programming once \( G(e^{i\omega}) \), \( c \), and \( k \) are given. In this case, \( G(e^{i\omega}) \) is given, but \( k \) and \( c \) are the ISOP parameters to be determined. To find values of \( k \) and \( c \), one may employ exhaustive search. Consider all possible \( (k, c) \) values. For each \( (k, c) \) pair, the optimization problem in (14) is solved by linear programming. Then a \( (k, c) \) pair associated with the optimal solution is chosen. This completes the design of both ISOP and programmable FIR filters. As in Section II-B, considering all \( k \) in the range given by (8) is not a difficult task. On the other hand, the search for a real value \( c \) is more difficult. A useful search range for \( c \) is given by

\[
c_0 < c < -2
\]  

(15)

where \( c_0 \) is the optimal \( c \) value obtained by solving the ISOP design problem in (9). A rationale for this range is as follows. The ISOP filter in this section should compensate additional passband droop caused by MHBF’s, as compared with the ISOP in Section II-B. The inequality in (15) follows from the observation that the slope of \( |P(e^{i\omega})| \) tends to increase as \( |c| \) is decreased (see Fig. 4). In the following section, it is observed that the time required for designing ISOP and programmable FIR filters by the proposed method is not excessive in practical applications.

**IV. DESIGN EXAMPLES**

This section presents two examples illustrating the procedure for designing the proposed decimation filter. In the first example, filter specifications in [9] are considered. In the second, specifications suitable for a programmable downconverter of the IS-95 mobile communication system [17] are specified and a decimation filter for IS-95 is designed. For multistandard communication, it is assumed that the input sampling frequency \( F_s \) can be adjusted so as to maintain an integer decimation factor \( D \). When this is impossible, an additional sampling rate converter such as the one in [18] is necessary. Throughout this section, the proposed architecture is compared with the one in [9], consisting of the CIC with \( R = 1 \), five halfband filters, and a programmable FIR filter. The architecture in [5] is excluded because its transition bandwidth is wider than the required transition bandwidths. Five MHBF’s (\( J = 5 \)) are employed having CSD coefficients that can be expressed as sums and differences of two powers-of-2 terms with 9-bit resolution. These MHBF’s were designed in cascade form by linear programming [11] for \( f_p \) in \{0.05, 0.075, 0.1, 0.125, 0.15\} and \( \delta_2 = 0.00001 \) [see (10)]. The magnitude response and the coefficients of these filters are shown in Fig. 9 and Table II, respectively. The implementation of MHBF’s in dedicated hardware is
not difficult. For example, MHBF5 in Table II, which is the most complex among the five, requires 19 adders and 13 shifters. This hardware complexity typically corresponds to a few multipliers.

**Example 1:** The specifications considered in designing a decimation filter in [9] (the Harris HSP50214) are as follows:

- **Sampling rate:** $F_s = 39$ Msp
- **Passband edge:** 90 kHz from the carrier
- **Stopband edge:** 115 kHz from the carrier
- **Desired decimation ratio:** $D = 72$. 

In normalized frequencies, these correspond to

| Passband: $f \in [0, 0.0023]$ 
| Stopband: $f \in [0.0029, 0.5]$.

The decimation filter designed in [9] consists of a CIC filter with $M = 18$, $L = 5$, and $R = 1$, two halfband filters ($m = 2$), and a 90-tap programmable FIR filter with even symmetric coefficients. The passband ripple and stopband attenuation that can be achieved by the decimation filter are

| Passband ripple: 0.18 dB 
| Stopband attenuation: 108 dB. 

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**TABLE II**

**The Powers-of-2 Coefficients for MHBF's**

| MHBF1 | $2^{-1}[2^{-2} + (2^{-1} - 2^{-7})z^{-1} + 2^{-2}z^{-2}][2^{-2} + 2^{-8} + (2^{-1} + 2^{-7})z^{-1} + (2^{-2} + 2^{-6})z^{-2}]$ |
| MHBF2 | $[2^{-2} + 2^{-4} + (2^{-1} - 2^{-4})z^{-1} + (2^{-2} + 2^{-4})z^{-2}][2^{-2} + 2^{-8} + (2^{-1} + 2^{-6})z^{-1} + (2^{-2} + 2^{-6})z^{-2}]$ |
| MHBF3 | $[2^{-2} + (2^{-1} - 2^{-5})z^{-1} + 2^{-2}z^{-2}][2^{-2} + 2^{-4} + (2^{-1} + 2^{-6})z^{-1} + (2^{-2} + 2^{-4})z^{-2}]$ |
| MHBF4 | $2^{-2}[2^{-1} - 2^{-4} + (2^{-1} + 2^{-3})z^{-1} + (2^{-1} - 2^{-4})z^{-2}][2^{-2} + 2^{-4} + (2^{-1} + 2^{-7})z^{-1} + (2^{-2} + 2^{-4})z^{-2}]$ |
| MHBF5 | $2^{-3}[2^{-1} + 2^{-3} + (2^{-1} + 2^{-2})z^{-1} + (2^{-1} + 2^{-3})z^{-2}][2^{-2} + 2^{-2} + 2^{-3}z^{-1} + 2^{-2}z^{-2}][1 + z^{-1}]$ |
Now, consider another decimation filter under the specifications in (16)–(18), designed following the procedure presented in Section III.

**Multistage Halfband Decimation Filter Design:** Since $D = 72 = 2^3 \times 9$ and, for $m = 3$, all of the MHBF’s in Fig. 9 satisfy (12), then $m_{\text{max}} = 3$, and there are four pairs of $(m, M)$ candidates. Among these, the pair $(m, M) = (3, 9)$ leads to a filter with minimal complexity. MHBF5 is used as a prefilter, and select MHBF1, MHBF2, and MHBF4 form three-stage $(m = 3)$ halfband decimation filters. This is because MHBF5 has a wider stopband than the others, and the cascade of MHBF1, MHBF2, and MHBF4 causes the least passband droop while providing 120 dB stopband attenuation.

**CIC Filter Design:** Since $D = 72$ and $m = 3$, the CIC decimation factor $M$ should be 9. Set $L = 4$ and $R = 1$. The CIC filter with these parameters provides 133.3 dB aliasing attenuation.

**ISOP and Programmable FIR Filter Design:** Given the CIC filter and the MHBF’s, (14) is solved using a linear programming package in [19]. The total design time in a personal computer with a Pentium 200 MHz processor is less than 2 h. The optimization results in ISOP parameters $(k, c) = (19, -2.481)$ and a 69–tap odd symmetric programmable FIR filter.

Fig. 10 shows the magnitude responses of the overall decimation filters designed by the proposed method and the method for the HSP50214. Computational complexities required for implementing the overall filters are compared in Table III. The proposed architecture reduced 21 multiplications at the expense of 15 additions and 15 delays.

**Example 2:** A desirable sampling frequency for IS-95 is $M_{\text{sps}} = 49.152$ Msp/s, which is 40 times the chip rate of 1.2288 Mchips/s. If the desired output rate of programmable FIR filtering is two times the chip rate, then set $D = 20$. The passband and stopband specifications of the overall decimation filter are determined based on those of a commercially available analog intermediate frequency filter used for IS-95 systems. Specifically, consider the filter in [20] with the following
Fig. 11. The magnitude response of the designed down converter for IS-95 system.

specifications:
- passband edge: 630 kHz from the carrier
- passband ripple: 0.7 dB
- stopband: 35 dB attenuation at 750 kHz from the carrier
- 50 dB attenuation at 900 kHz from the carrier. (19)

These specifications in normalized frequencies are
- passband: \( f \in [0, 0.0128] \)
- stopband: \( f \in [0.0153, 0.5] \)
- passband ripple: 0.2 dB
- stopband attenuation: 80 dB. (20)

The values 0.0128 and 0.0153 correspond to 630 and 750 kHz, respectively. The specifications in (20) are considerably more stringent than those in (19). Two decimation filters, the proposed and HSP50214-based filters, are designed under the specifications in (20). The procedure for designing these filters is summarized below.

The Proposed Filter Design: Since \( D = 20 = 2^4 \times 5 \) and, for \( m = 2 \), there exist two MHBF’s satisfying (12), then \( m_{\text{max}} = 2 \). Among the three \((m, M)\) pairs, \((1, 10)\) leads to a filter with minimal complexity. Again, use MHBF5 as a prefilter, and select MHBF4 to form a single-stage halfband decimation filter. The CIC filter with \( L = 5 \) and \( R = 1 \) is chosen. This CIC filter provides 83.9 dB aliasing attenuation. The optimization in (14) is solved as in Example 1. In this case, the design time was about 1 h. The optimum \((k, c)\) are given by \((4, -2.1516)\). The resulting programmable FIR filter has 50 taps, which are even-symmetric.

HSP50214-Based Design: Among the five halfband filters, select the fifth one which meets (12) for \( m = 1 \). The CIC filter with \( M = 10, L = 5, \) and \( R = 1 \) is chosen. The programmable FIR filter is designed by solving an optimization problem which is similar to the one in (14). The resulting programmable FIR filter has 65 taps, which are odd symmetric.

Fig. 11 shows the magnitude responses of the two overall decimation filters. From Table III, comparing their computational complexities, one can see that the proposed filter reduced 14 multiplications and four delays at the expense of ten additions.

V. CONCLUSION

An efficient CIC-based decimation filter employing an ISOP was proposed, and a design method was developed. It was observed that ISOP’s are very useful for reducing the computational complexity of decimation filters.

An interesting topic for further research is to find some other polynomials that can outperform ISOP’s. Examination of some higher order polynomials such as even-symmetric third-order polynomials would lead to another class of polynomials which is useful for CIC-based decimation filtering.

REFERENCES

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