Rate Optimization to Minimize Distortion for Source-Channel Coded H-BLAST with SIC Decoding

Jinho Choi and Jeongseok Ha

Abstract—As joint source and channel coding schemes, layered transmission schemes for a successive refinable source (SRS) are recently studied over quasi-static fading channels. In this letter, we consider spatial layered transmission for a SRS with successive interference cancellation (SIC) decoder over quasi-static multi-input multi-output (MIMO) fading channels. We show that if the distortion function can be factorized, the minimum average distortion can be obtained by individual rate optimization for each layer when zero-forcing (ZF) SIC receiver is employed over Rayleigh fading channels.

Index Terms—Source-channel coding, fading channels, successive refinement.

I. INTRODUCTION

Recen
tly, joint source and channel coding has been actively studied for quasi-static fading channels. It is shown in [1] that layered transmission for a successive refinable source (SRS) is an effective means to transmit analog sources over quasi-static fading channels when channel state information (CSI) is not available at the transmitter. At the receiver, the quality (or distortion) of the recovered signal depends on CSI and could be improved as the signal-to-noise ratio (SNR) increases (as in analog communications). Rate and power allocation is investigated to minimize average distortion in [2], [3].

For layered transmission, superposition coding is employed in [1], [3], [2]. When multiple antennas are used, various spatial layered transmission schemes are also available [4], [5]. Thus, it is natural to extend layered transmission for SRSs to multi-input multi-output (MIMO) systems [6]. To exploit spatial multiplexing gain, multiple independently encoded data streams can be transmitted through transmit antennas. If there are \( K \) transmit antennas, \( K \) data streams can be transmitted simultaneously. This layered transmission is called horizontal Bell Labs layered space-time (H-BLAST) architecture. At the receiver, successive interference cancellation (SIC) decoding can be employed to decode multiple data streams. A rate optimization to maximize throughput over quasi-static fading channels is considered with the zero forcing (ZF) SIC receiver [7], in which it is shown that the optimal rates can be found by a recursive approach without using a joint optimization technique.

In this letter, we consider a rate optimization for H-BLAST to transmit a SRS over quasi-static fading channels. We derive a recursive approach to find the optimal rates that minimize the average distortion for a certain distortion function.

II. SYSTEM MODEL

Suppose that the transmitter is equipped with \( K \) transmit antennas and the receiver is equipped with \( N \) receive antennas, where \( N \geq K \). We assume that the channel becomes static during a packet transmission. The channel matrix is denoted by \( \mathbf{H} \), which is a \( N \times K \) complex valued matrix. It is assumed that \( \mathbf{H} \) is known at the receiver, but unknown at the transmitter.

Suppose that a SRS can be decomposed into \( K \) sequences. Each (possibly compressed) sequence is encoded by an individual channel code. Denote by \( R_\ell \) the transmission rate for the \( \ell \)th sequence. The distortion of source \( \tilde{s}_\ell \) can be cancelled in the higher layers. This operation is recursively illustrated by

\[
\tilde{z}_\ell = \mathbf{Q}^H r_\ell = \mathbf{R} s_\ell + \tilde{n}_\ell,
\]

where \( \tilde{n}_\ell = \mathbf{Q}^H n_\ell \). Since \( \mathbf{Q} \) is unitary, the statistical properties of \( \tilde{n}_\ell \) are identical to those of \( n_\ell \). Layer \( \ell \) is called the lowest layer (thus, the highest layer becomes layer 1). For convenience, we omit the time index, \( t \). The \( K \)th element of \( z_\ell \), denoted by \( z_K \), can be decoded without any interference, because \( \mathbf{R} \) is upper triangular. Once the signals transmitted through lower layers are successfully decoded, these signals can be cancelled in the higher layers. This operation is repeated up to layer 1. Due to nulling, this receiver is called...
the ZF-SIC receiver. For better performance, the minimum mean square error (MMSE) based suppression can be used, which results in the MMSE-SIC receiver.

III. RATE OPTIMIZATION TO MINIMIZE DISTORTION

In this section, we consider the rate optimization to minimize the average distortion with the ZF-SIC receiver.

In [7], the rate optimization to maximize the average throughput has been considered when the ZF-SIC is used. From (2), if a Gaussian codebook is used for channel coding, with perfect SIC, the channel capacity for layer $k$ becomes $C_k = \log_2 (1 + \gamma_k |r_k|^2)$, where $r_k$ denotes the $(k,k)$th element of $\mathbf{R}$ and $\gamma_k = \frac{\sigma_k^2}{\sigma_n^2}$ is the SNR for layer $k$. Here, $\sigma_k^2 = E [|s_k|^2]$. The conditional channel outage probability for layer $k$ (provided that the SIC is successful) is given by

$$P_k = \Pr (R_k > C_k) = \Pr \left( |r_k|^2 < \frac{1}{\gamma_k} (2^{R_k} - 1) \right).$$

(3)

Let $F_k(z)$ denote the cumulative distribution function (cdf) of $z = |r_k|^2$. Then, it follows that $P_k = F_k \left( \frac{1}{\gamma_k} (2^{R_k} - 1) \right)$. If the $r_k$’s are independent of each other (thus, the $C_k$’s are also independent of each other), the average throughput with SIC decoding is given by

$$\mathcal{R}(\{R_k\}) = E \left[ \sum_{k=1}^{K} C_k \right] = R_K (1 - P_K) + R_{K-1} (1 - P_K) (1 - P_{K-1}) + \cdots + R_1 \prod_{q=1}^{K} (1 - P_q)$$

$$= \sum_{k=1}^{K} R_k \prod_{q=k}^{K} (1 - P_q).$$

(4)

In [7], a recursive approach to find the optimal rates that maximize the throughput is derived without using a joint optimization technique.

In this letter, under the same channel condition in (4), we consider a different problem – the minimization of average distortion for a SRS. In this case, distortion (rather than throughput) can be used as a performance index. If $s_q$, $q = K, K-1, \ldots, 1$, are decoded successfully, the distortion becomes $d_k = d(\alpha \sum_{q=k}^{K} R_q)$. Using a similar approach to find the average throughput in (4), the average distortion can be obtained as follows:

$$\mathcal{D}(\{R_k\}) = E[d_k] = d_{\text{max}} P_K + \sum_{k=1}^{K} B_k \prod_{q=k}^{K} (1 - P_q),$$

(5)

where $d_{\text{max}} = d(0)$,

$$B_k = d \left( \alpha \sum_{q=k}^{K} R_q \right) - d \left( \alpha \sum_{q=k+1}^{K} R_q \right), \quad k = 1, 2, \ldots, K-1,$$

and $B_K = d(\alpha R_K)$. Compared with (4), we can see that (5) has an additional term $d_{\text{max}} P_K$, which reflects the distortion when decoding at layer $K$ fails (thus, decoding for the other layers becomes impossible and the resulting distortion is the maximum distortion). For example, we can consider the case of $K = 2$, where the average distortion is given by

$$E[d_k] = d_{\text{max}} P_2 + d(\alpha R_2) \left( (1 - P_2) - (1 - P_1) (1 - P_2) \right) + d(\alpha (R_1 + R_2)) (1 - P_1) (1 - P_2)$$

$$= d_{\text{max}} P_2 + B_2 (1 - P_2) + B_1 (1 - P_1) (1 - P_2).$$

The rate optimization to minimize the expected distortion is given by

$$\{R_1^*, R_2^*, \ldots, R_K^*\} = \arg \min_{\{R_1, R_2, \ldots, R_K\}} \mathcal{D}(R_1, R_2, \ldots, R_K).$$

In finding $\{R_k^*\}$, to avoid a joint optimization technique, we consider a recursive approach using a one-dimensional optimization technique for each step under a certain condition.

Theorem 1: If the distortion function can be factorized as follows:

$$d \left( \alpha \sum_{q=k}^{K} R_q \right) = \prod_{q=k}^{K} d(\alpha R_q) (d(0) = d_{\text{max}} = 1),$$

(6)

we have $R_k^*$’s minimizing $\mathcal{D}$ in a recursive way such that

$$R_k^* = \arg \min_{R_k} \mathcal{D}_k(R_1^*, R_2^*, \ldots, R_{k-1}^*, R_k), k = 1, 2, \ldots, K-1,$$

(7)

and the rate for the bottom layer is from

$$R_K^* = \arg \min_{R_K} \mathcal{D}_k(R_1^*, R_2^*, \ldots, R_{K-1}^*, R_K) + P_K,$$

(8)

where $\mathcal{D}_k(R_1, R_2, \ldots, R_k)$ ($\mathcal{D}_k$ for short) is defined as

$$\mathcal{D}_k(R_1, R_2, \ldots, R_k) = \sum_{q=1}^{k} \tilde{B}_q^k \prod_{m=q}^{K} (1 - P_m), \quad k = 1, 2, \ldots, K,$$

$$\tilde{B}_q^k = B_q,$$

and

$$\tilde{B}_q^k = d \left( \alpha \sum_{m=q+1}^{K} R_m \right) - d \left( \alpha \sum_{m=q}^{K} R_m \right), \quad k < K.$$

Proof: For layer 1,

$$R_1^* = \arg \min_{R_1} \mathcal{D}_1(R_1) = \arg \min_{R_1} \min \left( \mathcal{D}_{1-1}(d(\alpha R_1) - 1) (1 - P_1) \right).$$

For layer $k < K$, we can express $\mathcal{D}_k$ as a recursive form:

$$\mathcal{D}_k = \sum_{q=1}^{k} \tilde{B}_q^k \prod_{m=q}^{K} (1 - P_m)$$

$$= \left( \sum_{q=1}^{k-1} \tilde{B}_q^{k-1} \prod_{m=q}^{K} (1 - P_m) \right) d(\alpha R_k) (1 - P_k)$$

$$+ \tilde{B}_k^k (1 - P_k)$$

$$= \mathcal{D}_{k-1} d(\alpha R_k) (1 - P_k) + \tilde{B}_k^k (1 - P_k),$$

where $\tilde{B}_q^k = \tilde{B}_q^{k-1} d(\alpha R_k)$ for $q < k$.

Suppose we have the set $\{R_1^*, R_2^*, \ldots, R_{k-1}^*\}$ that minimizes $\mathcal{D}_{k-1}$ (the minimum is denoted by $\mathcal{D}_{k-1}$):

$$\min_{R_1, R_2, \ldots, R_k} \mathcal{D}_k$$

$$= \min_{R_1, R_2, \ldots, R_k} \min \left( \mathcal{D}_{k-1} \right)$$

$$= \min_{R_1, R_2, \ldots, R_k} \left( \mathcal{D}_{k-1} (1 - P_k) + \tilde{B}_k^k (1 - P_k) \right)$$

$$= \min_{R_k} \mathcal{D}_{k-1} (d(\alpha R_k) (1 - P_k) + \tilde{B}_k^k (1 - P_k)).$$

(9)
In (9), we can see that $D_{k-1}$ and $R_k$ independently contribute to $D_k$, and the minimum of $D_{k-1}$ also the minimum of $D_k$. Finally, it is easy to confirm that $D_K(R_1, R_2, \ldots, R_K) + P_K$ is the same as $D(R_1, R_2, \ldots, R_K)$. Thus, the rates from the recursive steps in (7) and (8) also minimize $D$.

Based on Theorem 1, we can find the optimal rate recursively one-by-one without using a joint optimization technique. This can save the computational complexity. The key condition of Theorem 1 is (6). If the source is a unit-variance and zero-mean circular complex Gaussian random variable, it can be shown that [8]

$$d(R) = 2^{-R}.$$  \hspace{1cm} (10)

Thus, the condition in (6) holds. Furthermore, for other random variables, since the distortion function in (10) is an upper bound [8], the rate optimization in Theorem 1 becomes robust.

For numerical examples, assume that the distortion function of source is given in (10) and the elements of $H$ are independent zero-mean circular complex Gaussian random variables with unit variance. Then, the distribution function of $|r_k|^2$ is given by [9]:

$$f(z) = \frac{1}{(M+K-k)!} z^{M+K-k} e^{-z},$$

where $M = N - K$. Furthermore, the $y_k$’s are independent of each other and $P_k$ is given by $P_k = 1 - \frac{z^{k}}{(M+K-k)!} \Gamma(M + K + 1 - k, \frac{z}{\gamma_k}),$ where $\Gamma(n, x)$ is the upper incomplete gamma function, $\Gamma(n, x) = \int_{x}^{\infty} t^{n-1} e^{-t} dt$.

Fig. 1 shows numerical results when $K = 4$ and $N = 6$. It is assumed that $\gamma_k = \gamma$ for all $k$ as long as $R_k > 0$. If $R_k = 0$ (no transmission), no power is allocated to layer $k$ and this power is distributed to the other layers. For comparison purpose, the distortion over ergodic channels is also shown in Fig. 1 (a). If the length of codeword can be sufficiently long (in the case of delay constraint is not stringent) or CSI is known at the transmitter [10], the ergodic channel capacity, which is $E[\log_2 \det(I + \frac{1}{N_0} HR_n H^H)]$, where $R_n = E[ss^H]$, can be achieved. In Fig. 1 (a), we assume that $\frac{1}{N_0} R_n = \gamma I$. There is a significant gap between the distortions over quasi-static and ergodic channels. In Fig. 1 (b), the optimized rates for $K = 4$ layers are shown. Since a higher layer can exploit a better diversity gain, a higher rate can be assigned to minimize distortion (this behavior is similar to that in [7], in which the throughput is maximized).

IV. CONCLUDING REMARKS

We showed that H-BLAST can be used as a layered transmission scheme to transmit a SRS over quasi-static MIMO fading channels. A recursive approach was derived to find the optimal rates minimizing the average distortion for a factorizable distortion function. Although the optimal rates are available through a low complexity one-dimensional optimization technique, analytical solutions were not found. As a further research issue, we will consider asymptotic analysis and impact of diversity gain in optimizing rates based on [11].

REFERENCES