Electrohydrodynamics and electrorotation of a drop with fluid less conductive than that of the ambient fluid

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In this article, we investigated the electrohydrodynamic responses of a deformable fluid drop in another immiscible fluid under the action of a dc electric field. Both the ambient and drop fluids considered here were incompressible Newtonian and all of the drop phases were less conductive than their ambient fluids. Under these circumstances, the drops experienced the so-called electrorotation owing to the reverse dipole generated by the external electric field when the electric field strength exceeded a certain threshold value. The experimental observation showed that the threshold electric field strength was dependent on the drop size as well as the viscosity ratio. Also noted was the effect of the electrorotation on both the deformation behavior and the mode of the drop breakup. Specifically, we determined the critical electric capillary number beyond which the steady-state drop shape did not exist and the drop eventually broke up. Finally, the validity of Taylor’s leaky dielectric theory was discussed in the presence of the electrorotation of the drop.

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I. INTRODUCTION

Electrohydrodynamics describes the fluid flows induced by the external electric field. There are many important industrial and technical applications related to the electrohydrodynamic responses of the fluids. The fluid motions created by the electric field are responsible for the enhancement of heat or mass transfer rate, which is a crucial factor required for many processes such as electric field-enhanced liquid–liquid extraction and emulsion phase contactor. This electric field has been shown very effective in the containerless material processes such as crystal growth in microgravity condition. One of the promising applications of electrohydrodynamic phenomena can be also found in biophysical processes. It has been reported that the electric field has distinctive advantages in cell fission compared with conventional heat treatment because the process can proceed at low temperatures. The rotational motion of particles in an electric field has been also a subject under investigation. This provides a fundamental knowledge on the applications such as electrorotational spectroscopy.

In general, a drop suspended in an immiscible fluid is influenced by the externally applied electric field in many ways. Due to the presence of the drop, the electric field is disturbed around the drop, and thereby, the electric stress field is discontinuous across the fluid–fluid interface. Taylor’s well-known leaky dielectric theory describes the deformation behavior of the drop in this situation at least qualitatively. When both the drop and continuous phases are leaky dielectrics, free charges appear at the drop interface. In a uniform electric field, the charge distributions on the two hemispheres of the drop are antisymmetric so that the net surface charge is zero. The action of the electric field on these charges sets the fluids in motion and generates toroidal circulation flow patterns inside and outside the droplet. Meanwhile, when the two contiguous fluids are perfect dielectrics, the fluids remain motionless under a steady dc field. Furthermore, the external electric field leads to the drop deformation and eventually to the drop breakup as the field strength increases. If the electric field strength is lower than its critical value, the normal components of electric stresses can be mechanically balanced by the interfacial tension plus the normal components of viscous stresses associated with the fluid motions. The interface must then be deformed from its initial spherical shape. On the other hand, when the electric field strength exceeds a certain critical value, the drop cannot sustain its steady shape. Eventually, the drop is fragmented into smaller droplets.

During the past three decades, a lot of experimental and computational works elucidating the behavior of the electrically suspended drop have become available in the literature. As a result, we can predict accurately the behavior of a conducting drop immersed in an insulating medium. However, for a less conducting drop suspended in a more conducting ambient fluid, only a scarce attention has been paid. From an experimental point of view, this is partially due to the fact that the dielectric breakdown occurs easily when the medium is not highly insulating. Allan and Mason observed that the less conducting drop became flatten under the action of electric field and the toroidal shape was formed before the burst of the drop. Torza, Cox, and Mason reported the descriptive data on the deformation and burst of a less conducting drop. As predicted by Taylor, the drop deformed into an oblate spheroid in spite of the presence of electric field. The oblate-type deformation was due to the action of the biaxial strain-
ing motion of the fluids inside and outside the drop. The burst behavior was quite different from that of a more conducting drop suspended in a less conducting medium. Vizika and Saville conducted careful experiments to examine the validity of Taylor’s theory in a uniform or time-dependent electric field, and, more recently, Ha and Yang studied the deformation of a drop contaminated by nonionic surfactant. Unfortunately, however, these studies were restricted in the limit of small deformations. Moreover, in all of the aforementioned studies, the effect of the drop rotation induced by electric field has not been considered. Meanwhile, Krause mentioned studies, the effect of the drop rotation induced by limit of small deformations. Moreover, in all of the aforementioned studies, the effect of the drop rotation induced by electric field has not been considered. Meanwhile, Krause and Chandratreya showed that the rotational motion of the drop can alter the deformation behavior of the drop considerably. According to Krause and Chandratreya, the electrorotation can suppress the deformation, especially perpendicular to the applied field direction. This is probably due to the fact that the rotation of the droplet interferes in the electrohydrodynamic flow along the interface between the droplet and the continuous phase.

The electrorotation of heterogeneous spherical objects such as biological cells in a uniform ac electric field has been observed much more frequently than in a uniform dc electric field. The electric-field-induced rotation was attributed to the existence of a phase lag between the applied field and the induced dipole moments in the spheres. This phase lag causes neighboring particles to exert on each other torques that lead to the spinning of the spherical particles. The electrorotation of biological cells may be also induced by either deposition of charges on the cell surfaces or interaction of the alternating field with the oscillation of dipoles within the cells, see Pohl and Crane. If two cells are close to each other, each is affected by the neighboring dipole field as well as by the external ac electric field. Under these circumstances, one cell will exert torque on the other and vice versa, leading to rotation whose velocity depends on the relative position of the cells.

In this study, we consider experimentally the electrorotation of a less conducting drop suspended in a more conducting fluid and its effect on the deformation and burst behavior of the drop under the action of dc electric field. In the following sections, we will introduce briefly the well-known theory on the electrorotation of a rigid sphere in a dc electric field and then discuss the deviation from the theory which results either from nonspherical, oblate shape of the deformed drop or from the dissipation of rotational energy due to the viscous flow inside the drop. The failure of Taylor’s leaky dielectric theory will be discussed as well.

II. ELECTROROTATION OF A RIGID SPHERE

As early as 1896, Quincke reported that small solid spherical particles immersed in liquids would rotate spontaneously when subjected to an electrostatic field. The electric-field-induced rotation occurs when the electric field strength exceeds a certain threshold value. In addition, no rotation is observed unless the charge relaxation times of the particle and the ambient liquid meet a certain condition. This can be explained quite easily by considering the surface charge developed on the particle surface. Figures 1(a) and 1(b) illustrate the polarity of the surface charge for the two distinctively different cases; namely, $\tau_2 < \tau_1$ and $\tau_2 > \tau_1$ for the charge relaxation times,

$$\tau_1 = \frac{\varepsilon_1}{\sigma_1}, \quad \tau_2 = \frac{\varepsilon_2}{\sigma_2},$$

where $\varepsilon$ and $\sigma$ are the electrical permittivity and conductivity and the subscripts 1 and 2 denote the continuous liquid phase and the dispersed particle phase, respectively. When the charge relaxation time of the particle phase is shorter than that of the liquid phase (i.e., $\tau_2 < \tau_1$), the dipole moment $p$ is oriented in a stable configuration. If the dipole turns aside by rotation, the induced electric torque

$$T' = p \times E_0,$$

tends to restore the dipole to its stable configuration so that it is aligned again to the electric field $E_0$. On the other hand, if $\tau_2 > \tau_1$, the dipole moment $p$ itself is reversed, and any small orientational displacement of the particle produces a torque which tends to increase the displacement further.

The resulting expression for the relationship between the induced torque $T'$ and the angular velocity $\Omega$ is given by

$$T' = \frac{6 \pi \varepsilon a^3 E_0^2 (1 - \tau_1 / \tau_2) \Omega \tau_r}{(1 + 2 \varepsilon_1 / \varepsilon_2)(1 + 2 \sigma_2 / 2 \sigma_1)(1 + \Omega^2 \tau_r^2)},$$

where $a$ is the radius of the sphere and $\tau_r$ is the Maxwell–Wagner interfacial polarization relaxation time defined by

$$\tau_r = \frac{\sigma_2 + 2 \sigma_1}{\sigma_2 + 2 \sigma_1}.$$

Expression (3) for the torque induced by the electric field on a leaky dielectric sphere immersed in another leaky dielectric liquid can be readily verified by integrating the electrical surface moment relative to the center of the sphere, which is calculated from the Maxwell stress tensor. Since we neglect the inertial contributions to the fluid motions, the rotation of a sphere in the electrostatic field is restrained only by the viscous torque $T'$ imparted to a rotating sphere by a surrounding viscous liquid, that is,

$$T' = -8 \pi \mu a^3 \Omega,$$
where $\mu_1$ is the viscosity of the ambient fluid. Then, the threshold value $E^*_0$ of the electric field strength above which the sphere will continue to rotate in a dc electric field can be readily evaluated through the marginal stability condition,\textsuperscript{21} that is,

$$ E^*_0 = \sqrt{\frac{4\mu_1}{3\epsilon_1(1-\tau_1/\tau_2)\tau_r} \left[ 1 + \frac{2\epsilon_1}{\epsilon_2} \left( 1 + \frac{\sigma_2}{2\sigma_1} \right) \right]}. $$

(5)

One thing noteworthy is that the threshold value $E^*_0$ of the electric field strength is independent of the size $a$ of the dispersed particle. Although the outlined results were derived for the rigid spherical particles, they are expected to provide a qualitative description for the electrorotation of a fluid drop in an electric field.

From now on, let us consider the deformation of a drop in an electric field. It is well-known that a less conducting drop suspended in a more conducting drop experiences an oblate-type deformation and the degree of drop deformation is a function of the radius of the undeformed spherical drop. Conventionally, the degree of drop deformation is defined by

$$ D = \frac{L - B}{L + B}, $$

in which $L$ and $B$ are the length of semiaxis parallel and perpendicular to the electric field, respectively. Then, Taylor’s leaky dielectric theory\textsuperscript{8} predicts that the degree of drop deformation, $D$, in this situation, becomes

$$ D = \frac{9a \epsilon_1 E^2_0}{16\gamma(2 + \sigma_2/\sigma_1)^2\Phi}, $$

(6)

where $\gamma$ is the interfacial tension between the drop and its ambient fluid. In this case, the subscript 2 stands for the drop phase. The scalar function $\Phi$, which is the so-called Taylor’s discriminating function, is given by

$$ \Phi = \left[ \frac{\sigma_2^2}{\sigma_1} + 1 \right] - \frac{2\epsilon_2}{\epsilon_1} + \frac{3}{\epsilon_1}(\sigma_2^2/\sigma_1 - \epsilon_2/\epsilon_1) - \frac{2\mu_1 + 3\mu_2}{5\mu_1 + 5\mu_2}. $$

When $D>0$, the drop deforms into a prolate spheroid whereas $D<0$ corresponds to the oblate-type deformation. Expression (6) for the degree of deformation contains one dimensionless group, the so-called electric capillary number, $C_e = \frac{a \epsilon_1 E^2_0}{\gamma}$, describing the relative strength of the electric force to the restoring interfacial force. Thus, the degrees of drop deformation can be plotted into a single master curve as a function of the electric capillary number for given ratios of the viscosities, permittivities, and conductivities of the drop and continuous phases. It should be noted that the linear relationship of (6) between $D$ and $E^2_0$ is a consequence of asymptotic expansion procedure at small deformations. From the stability analysis for a steady-state drop shape, we can predict a critical electric capillary number at which the loss of stability of a drop shape occurs. It has been shown that, in case of a more conducting drop immersed in an insulating medium, the drop is elongated continuously and eventually fragmented above this critical point. However, for a less conducting drop, relatively little work has been done on the drop breakup via the oblate-type deformation.

### III. EXPERIMENTAL METHODS

The castor oil purchased from Aldrich Chemical Co. was used as a suspending medium without further treatment. Its viscosity, dielectric constant and conductivity were 0.75 Pa s, 3.8, and $3.9 \times 10^{-11}$ $\Omega^{-1}$ m$^{-1}$, respectively. Therefore, the charge relaxation time is 0.88 s, which is estimated from the definition of (1). In order to explore the effect of the viscosity ratio, $\lambda(=\mu_2/\mu_1)$, between the drop phase and its ambient fluid, we prepared six different grades of silicone oils with a wide range of the shear viscosity for the dispersed drop phase. The flow cell was made by transparent acrylic resin in rectangular shape and two vertical copper plates were used as electrodes. The gap distance between the electrodes was 3.5 cm. The space between the electrodes was occupied by the ambient fluid of castor oil. After a small volume of the silicone oil was deployed by a micropipette as a drop in the middle of flow cell, the electric field was applied by a high voltage dc power supply (Glassman High Voltage Inc., Series EH).

The onset of the electrorotation was detected by optical microscope equipped with a CCD camera. If necessary, the steady-state shape or burst behavior was recorded by a video cassette recorder and captured by a frame grabbing board during the replay of recorded images. In order to visualize the fluid flow inside the drop, trace of aluminum powder was dispersed in silicone oils. In a moderate strength of electric field, toroidal circulating flow was generated as predicted by Taylor.\textsuperscript{8} When the electrorotation occurred, the circulating flow patterns disappeared and simple rotating flow patterns were observed. We measured the threshold electric field strength for the occurrence of the electrorotation at which circulating flows were changed to rotational motions.

Accurate measurement of the interfacial tension is difficult. This is especially true when the two contiguous fluids are highly viscous and almost neutrally buoyant as considered here. In this case, the conventional techniques such as the pendant drop, ring or plate methods are not appropriate. In the present study, we followed a rather simple way to estimate the interfacial tension, based on the small deformation theory proposed by Taylor. Although there was little fluctuation, the interfacial tensions estimated from the small deformation theory were nearly constant around 4 mN/m, which was independent of the viscosity ratio $\lambda$. The measured value of the interfacial tension for each experimental run was used for calculating the electric capillary number or the degree of deformation defined by (6).

The conventional conductivity meters were not appropriate due to the extremely low conductivities of the oily fluids used here. Instead, the conductivities of the liquids used in experiments were measured by a parallel-plate fixture of 1 mm gap and a sensitive ammeter. After the field had been applied, the electric current was measured by the ammeter. The measured current was divided by the area of the plate to
give the current density. The conductivities were then obtained by dividing the current density by the electric field strength. The measured conductivities at various electric field strengths showed that the conductivities were nearly independent of the electric field strength $E_0$ in the range considered here. The permittivity of the castor oil was measured by a dielectric analyzer (TA Instrument) with a dip-type sensor. The permittivities of silicone oils were provided from the manufacturer. The rheological responses of the constituent fluids were measured by an ARES fluid rheometer (Rheometric Scientific Inc.) with or without applying the electric field. The viscosities of the fluids were not changed within the electric field strength range considered here. All of the measurements were carried out at room temperature $23 \pm 1 ^\circ$C. The material properties were included in Table I.

### IV. RESULTS AND DISCUSSION

In Figs. 2(a) and 2(b), the visualized flow patterns inside the less conducting drop are reproduced as typical examples for the stable and unstable responses to rotational disturbances, respectively. Also included in Figs. 2(c) and 2(d) for illustrative purpose are the schematic flow diagrams for the two distinctively different flow patterns created inside of the drop. The flow pattern reproduced in Figs. 2(a) or 2(c) has been predicted by the well-known Taylor’s leaky dielectric theory and visualized by many investigators. The strength of the circulating flow increases with the electric field strength and decreases with the viscosity of the drop phase. When the electric field strength exceeds a certain threshold electric field strength, the drop becomes unstable to small rotational perturbations. As a consequence, the axisymmetric circulating flow disappears and the drop begins to rotate. As can be seen from Figs. 2(b) or 2(d), the axis of symmetry of the oblate spheroid inclines and is not parallel to the direction of the electric field, which is in contrast to the case of the stable drop. Although the drop rotates continuously, and thus, has no steady-state shape, the degree of drop deformation remains constant within a certain range of the electric field strength. As we shall see shortly, further increase in the electric field induces “pseudodeformation” (named by Krause and Chandratreya), and finally leads to the drop breakup.

The time evolution of the electrorotation above a threshold value of the electric field is reproduced in Fig. 3. Within a short period of time after the inception of the electric field, the circulating flow is created inside the drop. At this stage, the axis of symmetry keeps parallel to the direction of the electric field. As time proceeds, the developed circulating flow diminishes and the axis of symmetry becomes oblique, which can be seen from frame 6 of Fig. 3. However,

![FIG. 2. Circulating and rotating flow patterns inside the silicone oil drop ($\mu=1000$ cSt). (a) $a=0.13$ cm, $E_0=2.86$ kV/cm; (b) $a=0.13$ cm, $E_0=3.54$ kV/cm. The circulating and rotating flow patterns are schematically illustrated in (c) and (d), respectively. One complete rotation takes approximately 9 s.](image1)

![FIG. 3. Time evolution of a rotating silicone oil drop ($\mu=1000$ cSt, $a=0.13$ cm) above a threshold electric field strength, $E_0=3.54$ kV/cm. The images are taken up at the time interval of 1 s after the electric field is turned on.](image2)

### Table I. Physical properties of the samples. $E_0^*$ is predicted from the theory.

<table>
<thead>
<tr>
<th>Experimental system</th>
<th>Drop phase</th>
<th>$\lambda$</th>
<th>$\varepsilon_2/\varepsilon_1$</th>
<th>$\sigma_2/\sigma_1$</th>
<th>$\tau_2$</th>
<th>$E_0^*$ [kV/cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp1</td>
<td>silicone oil ($\mu=50$ cSt)</td>
<td>0.07</td>
<td>0.71</td>
<td>0.16</td>
<td>3.84</td>
<td>2.70</td>
</tr>
<tr>
<td>Exp2</td>
<td>silicone oil ($\mu=300$ cSt)</td>
<td>0.39</td>
<td>0.72</td>
<td>0.16</td>
<td>3.91</td>
<td>2.70</td>
</tr>
<tr>
<td>Exp3</td>
<td>silicone oil ($\mu=600$ cSt)</td>
<td>0.78</td>
<td>0.72</td>
<td>0.15</td>
<td>4.14</td>
<td>2.66</td>
</tr>
<tr>
<td>Exp4</td>
<td>silicone oil ($\mu=1000$ cSt)</td>
<td>1.31</td>
<td>0.72</td>
<td>0.15</td>
<td>4.14</td>
<td>2.66</td>
</tr>
<tr>
<td>Exp5</td>
<td>silicone oil ($\mu=4000$ cSt)</td>
<td>5.37</td>
<td>0.72</td>
<td>0.15</td>
<td>4.14</td>
<td>2.66</td>
</tr>
<tr>
<td>Exp6</td>
<td>silicone oil ($\mu=10000$ cSt)</td>
<td>13.07</td>
<td>0.73</td>
<td>0.14</td>
<td>4.59</td>
<td>2.56</td>
</tr>
</tbody>
</table>
the degree of drop deformation is kept constant while the drop rotates continuously. The transition from circulating flow to simple rotational flow is due to the fact that the reverse dipole responsible for the electrorotation is not fully developed in a short period of time. However, it is not clear at present why the axis of symmetry is deviated from the electric field direction when the electrorotation is initiated.

In Figs. 4(a)–4(c), the degree of drop deformation is plotted as a function of the dimensionless electric capillary number. The data included in the plot are collected until the electrorotation occurs. The onset of the electrorotation is determined by examining both the flow patterns inside the drop and the orientation of the axis of symmetry. Thus, the final point at the largest capillary number for each drop corresponds to the limit point for the existence of steady-state shape observed before the onset of the electrorotation. Also included for comparison are the deformation curves calculated from the small deformation theory (6). It should be noted that the theoretical prediction based on the small deformation theory (6) is in good agreement with the experimental results for a highly viscous drop as shown in Fig. 4(c) for \( \lambda = 13.07 \). Meanwhile, the deviation from the experimental observation becomes larger for the less viscous drop [see Figs. 4(a) and 4(b) for \( \lambda = 0.07 \) and 0.39, respectively]. Also noted from Figs. 4(a)–4(c) is that either as the drop phase becomes highly viscous or as the drop size decreases, the electrorotation occurs at a lower electric field strength or equivalently at a lower electric capillary number. This observation will be discussed in detail shortly.

The discrepancy between the small deformation theory and the experimental deformation behavior displayed in Figs. 4(a)–4(c) can be explained by considering the charge convection effect. In fact, the leaky dielectric theory ignores the charge convection contributions completely. Obviously, however, this severe restriction may impose upon the leaky dielectric theory to predict an inaccurate deformation behavior. Recently, Feng\(^{25}\) considered the influence of the charge convection on the drop deformation and breakup. According
to his analysis, the charge convection tends to reduce the intensity of the electrohydrodynamic flow. Consequently, the drop experiencing the oblate-type deformation with the charge convection was predicted to be much less deformed than it would be in the absence of the charge convection. In contrast, for the prolate-type deformation, the charge convection increases the drop deformation. However, since the electrohydrodynamic flow is weak in the prolate-type deformation, the effect of the charge convection in increasing the degree of deformation is not significant. The relative significance of the charge convection effect is usually measured in terms of the electric Reynolds number, $Re_{E}$, defined as the ratio of the time scales of the charge convection by flow and the charge relaxation by Ohmic conduction. The electric Reynolds number is given by

$$Re_{E} = \frac{9(\varepsilon_1 + \varepsilon_2)}{10(\sigma_1 + \sigma_2)} \left| \frac{\varepsilon_1 \sigma_2}{\varepsilon_2 \sigma_1} - 1 \right| \varepsilon_2 E_0^2 \frac{E_0^2}{\mu_2}.$$ 

Therefore, as the drop phase becomes highly viscous relative to the ambient medium, the electric Reynolds number decreases, and consequently, the charge convection effect diminishes. For example, the electric Reynolds number $Re_{E}$ is 1.22 for the fluid pair of Exp1 ($\lambda = 0.07$) and $Re_{E} = 0.0004$ for the fluid pair of Exp6 ($\lambda = 13.07$), both at the electric field strength of 1 kV/cm. It is thus expected that the charge convection contribution to the drop deformation in Exp1 is more significant than in Exp6. This explains why the steady state drop deformation is deviated from the leaky dielectric theory especially when the drop phase is less viscous, see Figs. 4(a) and 4(c). Conventionally, the degree of drop deformation has been expressed as a function of $aE_0^2$ or the electric capillary number. By doing so, the master curves for the degree of deformation can be constructed from the experiments without controlling carefully the drop size and the electric field strength separately. It is thus a convenient way of presenting the experimental data when the charge convection effect is negligible. Strictly speaking, however, the charge convection effect appears to be dependent on the drop size $a$ when the degree of deformation, $D$, is plotted as a function of the electric capillary number, $C$. This is because the electric Reynolds number $Re_{E}$ is proportional to $E_0^2$ rather than $aE_0^2$. In the present study, the degree of drop deformation was plotted as a function of the electric capillary number for two different drop sizes, see Figs. 4(a)–4(c). As noted, the size dependence of the charge convection is not pronounced at least in our experimental systems. This is especially true in the limit of small deformations.

In Fig. 5, the threshold electric field strength $E_0^*\theta$ for the onset of the electrorotation is plotted as a function of the radius $a$ of the undeformed drop for various samples. It can be readily noted from Fig. 5 that the threshold electric field strength is a weakly decreasing function of the drop size for a fixed viscosity ratio $\lambda$. Rather, it is the viscosity ratio that produces much more significant influence on the threshold electric field strength. In particular, when the drop fluid is highly viscous, the electrorotation is induced more readily. The threshold electric field strengths predicted from the theory are included in Table I. It can be seen that the theory underestimates considerably the threshold electric field strengths of the present experimental runs except for the case in which a highly viscous or a relatively small drop is employed as in Exp6.

Now, let us discuss why the theoretical predictions for the less viscous drop deviate largely from the experimental observations for the onset of the electrorotation (5). At this point, it is noteworthy that the theory for the electrorotation outlined in the previous section has been developed for a rigid spherical particle in a dc electric field. As a matter of fact, the applicability of the aforementioned theory to our experimental systems should be somewhat limited. There is a considerable difference between the rotation of a solid particle and a liquid drop, since the fluid inside the drop is set in motion. Furthermore, a slightly conducting drop immersed in a highly conducting medium is deformed into an oblate spheroid under an electric field. As a consequence, both the electric and hydrodynamic torques exerted on the drop must be modified from (3) and (4), respectively. In this context, when either the drop phase is highly viscous or the drop is extremely small, the drop behaves much like a rigid body and the deviation from spherical shape is very small. Under these circumstances, the theory can predict accurately the electrorotation of the fluid drop. For a less viscous drop, however, the electric energy which tends to rotate the drop is probably transformed to viscous dissipation associated with the strong internal fluid motions. As a result, the higher electric energy is required to cause the electrorotation of a less viscous drop. It may be also suggested that the ellipsoidal shape of the drop is responsible for the slight decrease in the threshold electric field strength with increase in the drop size, which is displayed in Fig. 5. Recently, Morozov considered the rotation of a deformable drop in a viscous fluid under the action of the magnetic field. Their results showed that the viscosity of the drop phase is an important parameter on the rotation of the ellipsoidal droplet. It is obvious that a more rigorous theoretical analysis is required for explaining...
the electrorotation of a deformable ellipsoidal particle, which is out of the scope of this study.

Due to the electrorotation or vorticity of the electric-field-induced flow, the breakup behavior of a less conducting drop is quite different from that of a more conducting drop. The vorticity effect on the drop stability has been explored comprehensively for a drop subjected to the external flow fields in the absence of an electric field. An initially spherical drop reaches a limiting steady deformation when either the vorticity of the ambient mean flow field is considerable or the viscosity ratio is high. Thus, a highly viscous drop will not be broken up in a strong vorticity flow. Within a certain range of the viscosity ratio, the critical capillary number at which the drop breaks up has a minimum. On the other hand, in the two-dimensional irrotational flow, there exists always a critical capillary number in the entire range of the viscosity ratio. Furthermore, for a given viscosity ratio, the critical capillary number in an irrotational flow is much lower than in a simple shear flow. As mentioned earlier, the onset of the electrorotation does not imply the actual drop fragment. Instead, the drop rotates continuously with a finite degree of deformation even in a higher electric field strength above the threshold value. Much higher electric field is necessary for the actual drop breakup. In Fig. 6, the critical capillary number is plotted as a function of the viscosity ratio. Analogous to the drop subjected to a shearing flow, the critical capillary number decreases with the viscosity ratio, \( \lambda \), for less viscous drops, then appears to pass through a minimum near \( \lambda = 1 \) and increases for more viscous drops. The fact that a much stronger electric field is required to break up a less viscous drop is due to the pronounced charge convection effect, which tends to reduce the degree of drop deformation. On the other hand, for a highly viscous drop, the required stronger electric field for the drop breakup is due to the vorticity of the electric-field-induced flow driven from the electrorotation. However, although the fragment of a drop is greatly influenced by the electrorotation, the existence of a limiting steady deformation in the electric field is not obvious as the drop in shearing flow. This is primarily because the electric field exerts the so-called electric stresses on the drop surface, which are the additional deforming forces. Unlike a less conducting drop reported here, for the prolate-type deformation of a more conducting drop, the critical electric capillary number is nearly independent of the viscosity ratio. This is due to the fact that, for a more conducting drop, the electrically driven flow is too weak to reflect its hydrodynamic effect on the drop deformation and breakup.

Finally, in Figs. 7(a) and 7(b), the captured frames of a bursting drop above the critical capillary number are reproduced. General features of the drop fragmentation are not quite different from each experimental run. The drop becomes flatten instantaneously after the inception of a dc electric field. The folding of the flatten drop is observed. Rotation, folding, and unfolding occur simultaneously and the drop eventually fragments into several small droplets. Torza et al. also studied the burst of the less conducting drop in a steady and time-dependent electric fields. According to their study, time dependence of the electric field led to a somewhat different breakup mechanism. In a steady electric field, the drop was flattened into an oblate spheroid at first and lost its symmetrical shape. They suggested that this asymmetric breakup feature arose from the migration of the drop toward the electrode. In some experimental systems, they observed that the drop was flattened and then folded over and twisted. However, they did not mention the electrorotation of the drop. It is noted from Figs. 7(a) and 7(b) that the major axis of a deformed drop is initially perpendicular to the direction of the external electric field, but the fragmentation proceeds in the direction parallel to the electric field due to the rotation of the drop, which was not mentioned by Torza et al. It is also worth commenting that, in case of the prolate-type deformation of a more conducting drop, the transient deformation and breakup behavior above the critical electric capillary number can be described, in principle, simply by balancing the electric and viscous stresses and the interfacial tension. For the oblate-type deformation, however, the complicated phenomena of the electrorotation and the charge convection must be included in order to investigate the transient deformation behavior. It is thus obvious that further studies are needed for the complete understanding of the modes and mechanisms relevant to the drop burst.

Before summarizing our present results, it may be useful to mention the consequences of the electrorotation in some potential applications. Recently, our research group observed for the first time the negative viscosity effect in the electrorheological responses of the emulsion of a less conducting dispersed phase in a more conducting continuous phase. The negative viscosity effect becomes pronounced as either the dispersed phase becomes more viscous or as the conductivity of the continuous phase decreases. This tendency can be interpreted in terms of the electrorotation of the dispersed droplets considered here. It is well known that the negative

![Critical electric capillary number as a function of the viscosity ratio, \( \lambda \).](image-url)
field strength. The rotating dispersed phase acts like micro-sized motors and effectively reduces the friction between neighboring fluid layers.

V. CONCLUSION

The electric-field-induced rotation of deformable fluid drops is considered experimentally. For the entire experimental runs, the drop fluid used here is less conducting than the continuous phase. As expected from the well-known theory for the electrorotation of a rigid spherical particle in a dc electric field, there is a threshold electric field strength at which the drop begins to rotate. However, the theoretical prediction of a threshold electric field strength employed in this study is not satisfactory when both the electrically-driven viscous flow inside the drop and the degree of drop deformation are considerable. The threshold electric field strength increases as either the viscosity ratio increases or the drop size decreases. The validity of the leaky dielectric theory is also examined. When the viscosity ratio is much smaller than unity, the deviation from the theory is not negligible even within the small deformation limit. This is due to the charge convection effect. The critical electric capillary number, at which the drop actually breaks up, has a minimum at a certain range of the viscosity ratio. This result is consistent with that of a viscous drop subjected to an external shearing flow where the vorticity plays an important role.

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footnotes: