Throughput Analysis of a DS/SSMA Unslotted ALOHA System with Two User Classes

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Abstract—In this paper, we propose a direct-sequence spread spectrum multiple access (DS/SSMA) unslotted ALOHA system with two user classes and analyze the throughput of the proposed system. Mobile stations (MS’s) are divided into two classes according to payment or traffic characteristics, such as delay-intolerant and delay-tolerant. Different permission probabilities are assigned to each class so that the appropriate quality of service can be provided. We assume that the generation of class 1 and 2 messages are Poisson distributed and the message is divided into several packets before transmission. The system is modeled as a two-dimensional Markov chain under the assumption that the number of packets transmitted immediately by both user classes is geometrically distributed and the packet length is constant. We calculate the packet success probability and the throughput as a function of the signal-to-noise ratio (SNR) during packet transmission, considering the number of overlapped class 1 and 2 messages and the amount of their time overlap. Moreover, we show that the proposed system differentiates user packets according to class and maintains a high throughput even under heavy traffic conditions using access control based on the channel load.

I. INTRODUCTION

Recently, the code division multiple access (CDMA) method has attracted a great deal of attention among the many multiple access techniques since its capacity is greater than other access techniques in cellular systems. Present CDMA-based cellular systems have primarily been optimized for voice transmission. Wireless systems, however, must support multimedia services with a variety of quality-of-service requirements since the needs of data services, such as Internet web services, have experienced an exponential rate of increase in wireless mobile communications.

Many researchers have been directed at accommodating voice and data users in DS/SSMA ALOHA based systems [1],[2]. However, most studies have been restricted to slotted systems. In a DS/SSMA slotted ALOHA system, packet transmission is initiated only at the beginning of a slot, and the success of packet transmission depends on the amount of user interference within a slot. However, a packet in a DS/SSMA unslotted ALOHA system can be transmitted at any time. Hence, the unslotted system requires no synchronization and the level of user interference fluctuates during packet transmission. Because of this fluctuation, analysis of unslotted systems is more difficult compared with slotted systems.

In this paper, we propose a DS/SSMA unslotted ALOHA system with two user classes and analyze the throughput of the proposed system. We use an access control scheme based on channel load (the number of simultaneous transmissions). The hub station observes the channel load continuously for a certain period of time and estimates the average offered load. Since the system offered load is always less than the allowable maximum offered load using the access control scheme, the proposed system can maintain a high throughput even under heavy traffic conditions. In order to clarify the effect of access control, we analyze the throughput and evaluate the performance of a DS/SSMA unslotted ALOHA system with access control. This paper is organized as follows: In Section II, a system model is presented. The permission probability of each class is derived in Section III. In Section IV, a system analysis in view of the packet success probability and the throughput is described. In Section V, numerical results are provided, and concluding remarks are presented in Section VI.

II. SYSTEM DESCRIPTION

To evaluate the throughput performance of a DS/SSMA unslotted ALOHA system with two user classes, we consider a single-hop spread spectrum packet radio network with the following assumptions.

1. The packet radio network consists of an infinite number of independent MS’s and a hub station. A system to support two user classes is considered to have the following properties:
   - Class 1: The users of this class pay the service provider more money than the users of the other class because they are delay intolerant. An example data message of this class would be a packet voice or an emergency data message.
   - Class 2: The users of this class are satisfied with best effort service. They are delay tolerant. Data messages of this class would be generated by electronic mail or a file transfer service. Class 1 and 2 messages are generated by a Poisson distribution with arrival rates of \( \lambda_1 [\text{messages/sec}] \) and \( \lambda_2 [\text{messages/sec}] \), respectively. Generated messages are divided into packets. The number of packets in a message of each class is geometrically distributed with a mean of \( B_1 \) for class 1 and \( B_2 \) for class 2. The packet length is fixed to be \( L \) bits.
2. All MS’s share the same spreading code and each MS transmits packets at any bit. Also, the hub station can receive packet by properly resolving overlapped signals with random arrival times if there are sufficient time offsets among the received packets [3].
3. All transmitted packets are received with an equal power.
4. Packet bit errors are caused by multiple access interference and additive white Gaussian noise (AWGN). The bit error probability \( P_e(k) \) is given by [4], where \( k \) is the number of interfering packets.
III. ACCESS CONTROL ALGORITHM

The hub station observes the offered loads of class 1 and 2 users for a certain period. The hub station calculates the permission probability $P_{tr,i}$ of class $i$ users based on the observed offered load of class $i$ users, then broadcasts the permission probability to MS’s. An MS transmits a message with probability $P_{tr,1}$ or $P_{tr,2}$, or stops message transmission with probability $1 - P_{tr,1}$ or $1 - P_{tr,2}$ according to the class. The offered load $G$ usually varies slowly and, therefore, the offered load can be regarded as constant during the time period for the access procedure [5]. Taking account of this fact, we can estimate the offered load $G_i$ of class $i$ users based on the channel load of each class measured during $T_y$. Using the estimated offered load $g_i$, we calculate the permission probability $P_{tr,i}$ of class $i$ users. To achieve maximum system throughput, the total offered load must always be less than $G_{max}$, which is the total offered load giving the maximum system throughput in a DS/SSMA slotted ALOHA system. Also, to give priority to class 1 users, as the estimated total offered load $g$ increases to a value greater than $G_{max}$, the hub station immediately cancels the transmission of class 2 users and gradually controls the transmission of class 1 users. Hence, the permission probabilities are derived as follows:

$$P_{tr,1} = \begin{cases} 1 & \text{if } g_1 \leq G_{max} \\ G_{max}/g_1 & \text{if } g_1 > G_{max} \end{cases}$$

$$P_{tr,2} = \begin{cases} 1 & , \text{if } g_1 + g_2 \leq G_{max} \text{ and } g_1 \leq G_{max} \\ (G_{max} - g_1)/g_2 & \text{if } g_1 + g_2 > G_{max} \text{ and } g_1 \leq G_{max} \\ 0 & \text{if } g_1 + g_2 > G_{max} \text{ and } g_1 > G_{max} \end{cases}$$

In practice, we must deal with the problem of how to estimate the offered load of each class. To solve this problem, each packet has an information bit for its class. The hub station observes the offered load of each class using this information bit. If the hub station observes the offered load of each user class for a long period of time, the estimated offered load $g_i$ of class $i$ users is approximated to the real offered load $G_i$ of class $i$ users. That is, $g_i \approx G_i$ since $G_i$ usually varies slowly [5].

The total offered load $G$, defined as the average number of generated packets within a packet duration, can be expressed as the sum of the offered loads of class 1 and 2 users. That is $G = P_{tr,1} \cdot G_1 + P_{tr,2} \cdot G_2$, where $P_{tr,i}$ is the permission probability of class $i$ users and the offered load $G_i$ of class $i$ users is calculated as $G_i = \Lambda_i \cdot T_y \cdot B_i$, where $T_y$ is a packet duration (i.e., $T_y = L/R$). Also, $L$ [bits] is the length of a packet, and $R$ [bits/sec] is the data rate. The value $B_i$ is the average number of several continuous packets transmitted immediately by a class $i$ user. Since we assume that $B_i$, the number of continuous packets transmitted immediately by a class $i$ user, is geometrically distributed with a mean of $B_i$, the probability that $B_i$ is $x$ is given by

$$P_x(B_i = x) = p_i \cdot (1 - p_i)^{x-1}, \text{ where } p_i = 1/B_i.$$  

We assume that the number of several continuous packets transmitted immediately by a class $i$ user is less than equal to $B_{MAX,i}$.

IV. THROUGHPUT ANALYSIS

A. Transition of the Number of Interfering Messages

We analyze the throughput of a DS/SSMA slotted ALOHA system under a single cell system where the interference level varies during message transmission because MS’s attempt to transmit messages at any bit. To evaluate the packet success probability such that there are no bit errors in the packet received at the hub station, we suppose a “tagged” packet, as shown in Fig. 1, and, for simplicity, arrange other messages in order. This figure shows that the number of class 1 and 2 interfering messages varies during transmission of the tagged packet. Because generation of class $i$ messages is assumed to be Poisson distributed with an arrival rate of $\Lambda_i$, the probability $P_{b,i}(k)$ that $k$ messages are transmitted during the packet duration $T_y$ by class $i$ users is given by

$$P_{b,i}(k) = \frac{(\Lambda_i T_y)^k}{k!} e^{-\Lambda_i T_y}$$

Let the number of class 1 and 2 interfering messages be $m_1$ and $n_1$, respectively, at the beginning of transmission of the tagged packet. Now, we evaluate the probability $P_{r}(m_1, n_1)$ messages at time $\tau_1$ that $m_1$ class 1 and $n_1$ class 2 messages are observed at time $\tau_1$, as shown in Fig. 1. First, we calculate the probability $P_{r}(k_1)$ messages at time $\tau_1$ that $k_1$ class 1 messages are observed at time $\tau_1$. Here, we define index sets as follows:

- Let index set $T = \{ \cdots, (\tau - 1), \tau, (\tau + 1), \cdots \}$ denote the bit time axis where the interval between $\tau$ and $(\tau + 1)$ is the packet duration $T_y$ which is equivalent to $L$ bits. Also, let the interval $[\tau_i, (\tau_i + 1)]$ be `period $\tau$`.
- Consider an index set $A_i = \{a_1, a_2, \cdots, a_y, \cdots \}$. When observed at time $\tau_1$, the $y$ labeled arrival $a_y$ is the number of class $i$ messages that enters the hub station in the period $(\tau - y)$ and departs after time $\tau_1$. The symbol $y$ denotes the length of message, that is, the number of packets in a message.

Then,

$$P_{r}(a_y \text{ at time } \tau_1)$$

$$= P_r \left( \text{At time } \tau_1, \text{the number of class } i \text{ messages that enter in period } (\tau - y) \text{ and depart after time } \tau_1 = a_y \right)$$

$$= \sum_{k=a_y}^{\infty} \left( \begin{array}{c} k \\ a_y \end{array} \right) \cdot P_x(B_i \geq y)^{a_y} \cdot (1 - P_x(B_i \geq y))^{k-a_y} \cdot P_{b,i}(k)$$

Fig. 1. Fluctuation of interference levels during tagged packet transmission
where \( P_x(B_i \geq y) \), which is the probability that a class \( i \) user transmits \( y \) or more packets in a message immediately, is calculated as follows:

\[
P_x(B_i \geq y) = \sum_{x=y}^{\infty} p_i \cdot (1 - p_i)^{x-1}
\]

\[
= (1 - p_i)^{y-1}, \text{ for } y \geq 1.
\]  

(5)

For the above equation (4), the following items can be applied: 1) For class \( i \), the \( k \) messages arrive during the period \( (\tau - y) \) and depart after the beginning of period \( \tau \) (after time \( \tau_1 \)). 2) Only \( a_y \) messages among \( k \) messages are observed at time \( \tau_1 \).

We define a probability \( Pr(A_i \mid \text{at time } \tau_1) \) as the probability that the number of class \( i \) messages that enter into \( \{ \text{period } \tau - 1, \text{period } \tau - 2, \ldots, \text{period } \tau - y, \ldots \} \) and depart from the channel after time \( \tau_1 \) is equal to \( \{ a_1, a_2, \ldots, a_y, \ldots \} \). As packet arrivals are independently generated, the probability \( Pr(A_i \mid \text{at time } \tau_1) \) is obtained by multiplication of each \( Pr(a_y \mid \text{at time } \tau_1) \) as follows:

\[
Pr(A_i \mid \text{at time } \tau_1) = \prod_{y=1}^{B_{\text{MAX},i}} Pr_i(a_y \mid \text{at time } \tau_1)
\]  

(6)

Accordingly, the probability \( Pr_i(k_1 \text{ messages \ at time } \tau_1) \) that \( k_1 \) class \( i \) messages are observed at time \( \tau_1 \) is given by:

\[
Pr_i(k_1 \text{ messages \ at time } \tau_1) = \sum_{k \in U_1} Pr(A_i \mid \text{at time } \tau_1)
\]

\[
= P_{I_1}(k_1)
\]  

(7)

where \( U_1 \) is the set \( \{ \forall a_y \in A_i \mid \sum_{y=1}^{B_{\text{MAX},i}} a_y = k, a_y \geq 0 \} \).

As class 1 and 2 messages are independently generated, the joint probability \( Pr((m_1, n_1) \text{ messages \ at time } \tau_1) \) that \( m_1 \) class 1 and \( n_1 \) class 2 messages are observed at time \( \tau_1 \) is obtained by multiplication of \( Pr_1(m_1 \text{ messages \ at time } \tau_1) \) and \( Pr_2(n_1 \text{ messages \ at time } \tau_1) \):

\[
Pr((m_1, n_1) \text{ messages \ at time } \tau_1)
\]

\[
= Pr_1(m_1 \text{ messages \ at time } \tau_1) \cdot Pr_2(n_1 \text{ messages \ at time } \tau_1)
\]

\[
= P_I(m_1, n_1).
\]  

(8)

When the system state is defined as the number of interfering messages of each class, we consider the state transition during transmission of the tagged packet. Let the number of interfering messages at the \( j \)-th bit of the tagged packet be \( (m_j, n_j) \) where \( m_j \) and \( n_j \) are the number of class 1 and 2 messages at the \( j \)-th bit, respectively. Then, under the assumption that the bit duration is small, the number of interfering messages, 1) increases to \((m_j + 1, n_j)\) or \((m_j, n_j + 1)\), 2) decreases to \((m_j - 1, n_j)\) or \((m_j, n_j - 1)\), 3) remains the same during bit timing since the interference level varies bit by bit during tagged packet transmission. If the bit timing is \( \Delta t \), then the system can be modeled as a two dimensional Markov chain, as shown in Fig. 2.

Let \((m_1', n_1')\) be the number of messages at the first bit of the tagged packet. If \((m_1', n_1')\) messages among \((m_1, n_1)\) depart during the packet duration \( T_p \), as shown in Fig. 1, the average service times of \( m_1' \) and \( n_1' \) messages are \( T_p/m_1' \) and \( T_p/n_1' \), respectively. Therefore, the death rates of class 1 and 2 messages are derived as [6]

\[
\mu_1(m_1', n_1') = m_1'/T_p \quad \text{and} \quad \mu_2(m_1', n_1') = n_1'/T_p.
\]

(9)

Also, the birth rate \( \Lambda_i \) of class \( i \) messages is obtained as follows:

\[
\Lambda_i = \frac{G_i}{T_p \cdot B_i} \approx \frac{g_i}{T_p \cdot B_i}
\]

(10)

where \( G_i \) is the real offered load of class \( i \) users and \( g_i \) is the estimated offered load of class \( i \) users. We calculate the birth rate \( \Lambda_i \) from observation of parameters \( g_i \) and \( B_i \) at the hub station. The hub station continuously observes the offered load and the message length of class \( i \) users and estimates \( G_i \) and \( B_i \).

Accordingly, the conditional state transition probability from \((j-1)\)th bits to \( j \)-th bits is given by:

\[
q(m_j, n_j \mid m_{j-1}, n_{j-1}) = \begin{cases}
1 - Pr_1 \Lambda_1 \Delta t - \mu_1(m_1', n_1') \Delta t - Pr_2 \Lambda_2 \Delta t - \mu_2(m_1', n_1') \Delta t, & \text{if } m_j = m_{j-1}, n_j = n_{j-1} \\
\mu_1(m_1, n_1) \Delta t, & \text{if } m_j = m_{j-1} - 1, n_j = n_{j-1} \\
Pr_1 \Lambda_1 \Delta t, & \text{if } m_j = m_{j-1} + 1, n_j = n_{j-1} - 1 \\
\mu_2(m_1', n_1') \Delta t, & \text{if } m_j = m_{j-1}, n_j = n_{j-1} - 1 \\
Pr_2 \Lambda_2 \Delta t, & \text{if } m_j = m_{j-1} - 1, n_j = n_{j-1} + 1 \\
0, & \text{otherwise}
\end{cases}
\]

(11)

**B. Derivation of Packet Success Probability**

To calculate the packet success probability, we define a function \( f_j(m_j, n_j, m_1, n_1, m_1', n_1') \) as follows [6]:

1. The function \( f_j(m_j, n_j, m_1, n_1, m_1', n_1') \) is the probability that the tagged packet is successfully transmitted from the first
bit to the $(j - 1)$th bit, where $m_j$ and $n_j$ are the number of interfering class 1 and 2 messages at the $j$-th bit, respectively.

2. The values $m'_1$ and $n'_1$ are the number of messages that depart during the packet duration $T_p$ among $m_1$ and $n_1$ messages, respectively when the level of interference at the first bit is $(m_1, n_1)$.

We evaluate the function $f_j(m_j, n_j, m_1, n_1, m'_1, n'_1)$ recursively based on the Markovian property of $m_j$ and $n_j$. For $j = 1$, $f_1(m_j, n_j, m_1, n_1, m'_1, n'_1)$ is equal to $P_{r_1}(m_1, n_1, m'_1, n'_1)$, which is the probability that $m'_1$ messages depart during the packet duration $T_p$ among $m_1$ messages and $n'_1$ messages depart in the same duration among $n_1$ messages when the level of interference at the first bit is $(m_1, n_1)$. Hence, we have

$$f_j(m_j, n_j, m_1, n_1, m'_1, n'_1) = P_{r_1}(m'_1, n'_1)$$
$$= P_{r_1}(m_1, n_1, m'_1, n'_1)$$
$$= P_{r}(m_1, m'_1, n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$
$$= P_{r_1}(m_1, m'_1) \cdot P_{r_2}(n_1, n'_1)$$

where $P_{r}(m_1, m'_1) = 1 - P(B_1 = y) \cdot P(B_2 = y)$.

For $j = 1$, $f_1(m_j, n_j, m_1, n_1, m'_1, n'_1)$ is obtained by (8).

When $j$ is not the first bit of tagged packet, $f_j(m_j, n_j, m_1, n_1, m'_1, n'_1)$, the probability that a packet is successfully transmitted from the first bit to the $(j - 1)$-th bit, becomes the probability that there is no error at the $(j - 1)$-th bit in a packet successfully transmitted to the $(j - 2)$-th bit. Hence, for $j > 1$, $f_j(m_j, n_j, m_1, n_1, m'_1, n'_1)$ is calculated recursively as follows:

$$f_j(m_j, n_j, m_1, n_1, m'_1, n'_1) = \sum_{m_j, m'_1, n_j, n'_1} \left[ f_{j-1}(m_{j-1}, n_{j-1}, m_1, n_1, m'_1, n'_1) \cdot q(m_j, n_j, m_{j-1}, n_{j-1}) \cdot (1 - P_B(m_j + n_j)) \right]$$

Using $f_j(m_j, n_j, m_1, n_1, m'_1, n'_1)$, we calculate the packet success probability $Q_s$ recursively. Since the packet length $L$ is constant, the packet success probability is calculated by setting $j = L$ as follows:

$$Q_s = \sum_{m_L = 0}^{\infty} \sum_{n_L = 0}^{\infty} \sum_{m_1 = 0}^{\infty} \sum_{n_1 = 0}^{\infty} \left[ f_L(m_L, n_L, m_1, n_1, m'_1, n'_1) \cdot (1 - P_B(m_L + n_L)) \right]$$

On the other hand, the system throughput is defined as the average number of successful transmissions during packet duration $T_p$. Hence, the system throughput $S = G \cdot Q_s$ and the throughput $S_i$ of class $i$ users are obtained by $S_i = P_{s_i} \cdot G \cdot Q_s$.

V. NUMERICAL AND SIMULATION RESULTS

We compare numerical results with simulation results for an unslotted system with two user classes. Because the number of messages transmitted at the same time may be neglected, we assume that packet bit errors are caused only by multi-user interference. Also, for simplicity, the effect of additive white Gaussian noise is not considered. In this simulation, the data rate $R = 0.6$ kbps, packet length $L = 512$ bits, and spreading factor $N = 30$ are assumed. The data traffic model of class 1 users is assumed to be a packet voice model with an average message length of 448 bytes ($B_1 = 7$) [7] and the data traffic model of class 2 users is assumed to have variable length with an average message length of 320 bytes, based on web traffic ($B_2 = 5$) [8]. Also, the spread factor $N = 30$ and packet length $L = 512$ (bits) are assumed.

Fig. 3 and Fig. 4 show the system throughput ($S$) versus the message arrival rate per packet duration ($\alpha = \alpha_1 = \alpha_2$). Here, the symbol $\alpha_i$ indicates the arrival rate of class $i$ messages per packet duration $T_p$ ($\alpha_1 = \lambda_1^{-T_p}$). When the hub station does not control MS access, $S$ increases with $\alpha$, but eventually decreases as $\alpha$ becomes larger. Thus, the system throughput decreases as the total offered load $G$ becomes larger than the maximum allowable offered load $G_{max}$. The throughput of class 1 users ($S_1$) is higher than that of class 2 users since the average length of class 1 messages ($B_1 = 7$) is longer than the length of class 2 messages ($B_2 = 5$). When the hub station controls MS access, the throughput of class 1 and 2 users are shown in Fig. 4. Since the hub station begins to control MS transmissions at the point $A$ where the total offered load becomes $G_{max}$, the curve is kinked at the point $A$. As the total offered load becomes larger, the hub station suppresses the transmissions of class 2 users so that the total offered load does not become greater than $G_{max}$. When the offered load of class 1 users becomes $G_{max}$ (i.e., reaches the point $B$), the hub station controls the transmission of class 1 users and rejects the transmission of class 2 users. The total offered load, therefore, is always less than $G_{max}$ and the maximum system throughput is maintained even under heavy traffic conditions.

When the arrival rate of class 1 messages is constant ($\alpha_1 = 0.6$), the throughput of class 2 users ($S_2$) versus the arrival rate of class 2 messages ($\alpha_2$) is shown in Fig. 5 and Fig. 6. Under heavy traffic conditions, the total throughput decreases as the total offered load becomes greater than $G_{max}$ when the hub station does not control MS transmission, as shown in Fig. 5. The throughput of each class, however, remains constant under heavy traffic conditions when the hub station broadcasts the permission probability of each class, as shown in Fig. 6.

VI. CONCLUSIONS

In the near future, many mobile users will require services with different qualities-of-service and the number of users will also dramatically increase. We have proposed a DS/SSMA unslotted ALOHA system with two user classes and analyzed the throughput of the proposed system. An access control scheme based on the channel load of each class is used. As the transmission of each user packet is controlled by the permission probability from the hub station, the proposed system maintains a
high throughput even under heavy traffic conditions and differentiates user packets according to its class. This system analysis, while limited to two classes herein, can be applied without modification to DS/SSMA unslotted ALOHA systems with multiple classes.

REFERENCES


