Effects of Distance Between Pads on the Inlet Pressure Build-Up on Pad Bearings

Full Navier-Stokes equations are solved numerically for a cavity region between two consecutive pads and a parallel lubricating film. Numerical solutions are obtained for a wide range of Reynolds number and various values of a distance between pads. Numerical results show that the inlet pressure build-up is significantly affected by Reynolds number and the distance between two adjacent pads. A new formula is derived of loss coefficient with Reynolds number and a distance factor, for using it in an extended Bernoulli equation, on the basis of numerical results. Experiments are conducted to investigate the validity of the formula of loss coefficient proposed by authors.

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Introduction

Under operating conditions of relatively high Reynolds number, the pressure build-up at the bearing entrance must be considered as an inlet pressure boundary condition in the performance analysis of pad bearings with a discontinuous lubricating film. It has been pointed out by many researchers [1–6] that the inlet pressure build-up can have a large influence on the bearing performance depending on the operating conditions. Therefore, it is very important to estimate precisely the inlet pressure build-up in the performance analysis of pad bearings. The flow models to evaluate the inlet pressure build-up have been developed by some workers (Constantinescu and co-workers [7]; Tichy and Chen [8]; Tipei [9,10]; Pan [11]; Mori and Mori [4]). These simplified flow models are very easy to use, but the correctness of the inlet pressure obtained from these models has not been verified completely.

In order to evaluate more reasonable inlet pressure build-up, several workers (Rhim and Tichy [13,14]; Mori and co-workers [15]), including Heckelman and Ettles [12], attempted to solve the full Navier-Stokes equation by a numerical analysis for a whole region including the cavity region between consecutive pads and the lubricating film. Through these numerical approaches, the inlet pressure can be estimated more accurately than by the simplified flow models. But using this numerical approach, which is solving the full Navier-Stokes equation for the whole region, is not realistic in the performance analysis of pad bearings because it requires so much computational effort.

The purpose of the present paper is to propose a new model combining the simplified flow model and the full numerical approach. Therefore, by this new model, the inlet pressure build-up will be obtained much easier than the full numerical approaches and more exactly than the simplified flow models. The present new model is the extended Bernoulli equation with a new loss formula. The new loss formula is developed from the solutions of the full Navier-Stokes equations for whole region including the parallel lubricating film and the cavity region between pads. Since the loss formula includes the Reynolds number and the distance between pads, the inlet pressure can be evaluated including the effects of the distance between pads as well as the operating conditions such as the sliding velocity, the viscosity of lubricant, the film thickness and so on. The present results show that the effect of the distance between pads on the inlet pressure is significant. Experiments are also performed and the results are compared with the theoretical predictions by the present model.

Numerical Analysis

Pad bearings must have a discontinuous lubricating film due to mainly a cavity between pads for the lubricating oil access, as shown in Fig. 1. The flow of the lubricating oil in the cavity is gradually developed along the sliding direction by the friction force on the runner surface. The developed flow will impact on the bearing entrance and then the pressure near the entrance will be built up more than the peripheral pressure (generally the ambient pressure). The pressure build-up at the bearing entrance will have an effect on the bearing performance such as seen in the load carrying capacity, the flow rate, the frictional loss, the dynamic coefficients, the pressure center (or the optimum pivot position in the tilting pad bearings) and so on. The flow field for the numerical analysis of the full Navier-Stokes equations is assumed as Fig. 2 based on the control volume of Fig. 1.

The flow in the cavity region of a finite width pad may be very complex in practice. However, the flow in the mid plane of the finite width pad may be similar with the flow of infinite width pad. Therefore, the inlet pressure build-up in the mid plane of the pad can be evaluated from two dimensional numerical analysis. And the inlet pressure distribution along the width of the pad can be decided by assuming as a proper function. The flow in the cavity region and the bearing film is assumed to be steady, laminar, two-dimensional, isothermal, and incompressible. The two dimensional Navier-Stokes equations and the conservation of mass are

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

\[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]

\[ \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]

\[ \frac{\partial (u \rho)}{\partial x} + \frac{\partial (v \rho)}{\partial y} = 0 \]

\[ \frac{\partial (u \rho h)}{\partial x} + \frac{\partial (v \rho h)}{\partial y} = 0 \]

\[ \frac{\partial (u \rho c_v T)}{\partial x} + \frac{\partial (v \rho c_v T)}{\partial y} = 0 \]

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Fig. 2 Representation of numerical solution domain

Fig. 3 Inlet pressure coefficient

Fig. 4 A simplified flow model for the extended Bernoulli equation

Fig. 5 Loss coefficient

Table 1 Coefficient values of Eq. (8) for distance factor

<table>
<thead>
<tr>
<th>λ</th>
<th>C₁</th>
<th>C₂</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5.766</td>
<td>0.615</td>
<td>1.079</td>
</tr>
<tr>
<td>40</td>
<td>5.911</td>
<td>0.592</td>
<td>1.020</td>
</tr>
<tr>
<td>100</td>
<td>6.108</td>
<td>0.562</td>
<td>0.945</td>
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<td>0.540</td>
<td>0.894</td>
</tr>
<tr>
<td>1000</td>
<td>6.633</td>
<td>0.492</td>
<td>0.783</td>
</tr>
</tbody>
</table>

Fig. 6 Comparison between loss coefficients by numerical analysis and by Eqs. (8) and (9)
Numerical Results and Discussions

and non-dimensional form of the equations are

\begin{align}
\frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} &= - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \\
\frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} &= - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0
\end{align}

and the boundary conditions are

\begin{align}
U(X,0) &= 1, \quad V(X,0) = 0, \\
U(X,H_g^s + h^s) &= V(X,H_g^s + h^s) = 0, \\
U(0,Y) &= U(1 + L_g^s, Y) = 0 \quad \text{for} \quad h^s \leq Y \leq H_g^s + h^s, \\
V(0,Y) &= V(1 + L_g^s, Y) = 0 \quad \text{for} \quad h^s \leq Y \leq H_g^s + h^s, \\
P^s(0,Y) &= P^s(1 + L_g^s, Y).
\end{align}

The velocity distributions of the exit of the previous pad and the present pad as shown in Fig. 2 must be the same but are not known prior to the analysis. But the velocity can be assumed as follows for a lubricating film.

\begin{align}
U(0,Y) &= U(1 + L_g^s, Y) = \frac{4}{\nu} \frac{\partial p^*}{\partial x} \left( \frac{Y}{h^s} - 1 \right) \\
&\quad + \left( 1 - \frac{Y}{h^s} \right), \quad 0 \leq Y \leq h^s, \\
V(0,Y) &= V(1 + L_g^s, Y) = 0
\end{align}

If the full Navier-Stokes equations are required for the lubricating film, the velocity at the bearing entrance may be not described by Eq. (5) because of the inertia effects at higher Reynolds number, and the periodic boundary condition cannot be imposed. The pressure gradient at the exit of the present pad in the Eq. (5) is found by the Newton-Raphson method, and the pressure periodic condition is achieved by following equation:

\begin{align}
\frac{|P^s(0,Y) - P^s(1 + L_g^s, Y)|}{P^*_{\text{max}}} &\leq 10^{-4}, \quad 0 \leq Y \leq h^s,
\end{align}

where $P^*_{\text{max}}$ is a maximum pressure in the lubricating film.

Numerical Results and Discussions

The ratio of the inlet pressure to the dynamic pressure ($\rho U^2/2$), i.e., the inlet pressure factor is shown in Fig. 3. It shows that the inlet pressure factor is apparently varied with the Reynolds number and the distance between the pads. With a lower Reynolds number, the inlet pressure factor can be much more than unity. The inlet pressure factor increase as the distance between the pads is longer. The effect of the distance between the pads on the inlet pressure factor becomes stronger at higher Reynolds number. And the bearing film thickness also gives the effect on the inlet pressure factor. The inlet pressure factor is higher as the film thickness is smaller.

The simplest way able to evaluate the inlet pressure is to apply the extended Bernoulli equation to the flow model as shown in Fig. 4. The extended Bernoulli equation is

\begin{align}
\rho u_u + \rho u_v + \frac{\partial p}{\partial y} &= - \frac{\partial p^*}{\partial x} + \mu \left( \frac{\partial u^2}{\partial x^2} + \frac{\partial u^2}{\partial y^2} \right), \\
\rho u_u + \rho u_v + \frac{\partial p}{\partial y} &= - \frac{\partial p^*}{\partial y} + \mu \left( \frac{\partial v^2}{\partial x^2} + \frac{\partial v^2}{\partial y^2} \right), \\
\rho u_u + \rho u_v + \frac{\partial p}{\partial y} &= 0
\end{align}

and non-dimensional form of the equations are

\begin{align}
\frac{\partial U}{\partial X} + \nu \frac{\partial V}{\partial Y} &= - \frac{1}{2} \frac{\partial p^*}{\partial X} + \frac{h^s}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \\
\frac{\partial V}{\partial X} + \nu \frac{\partial V}{\partial Y} &= - \frac{1}{2} \frac{\partial p^*}{\partial Y} + \frac{h^s}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right), \\
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0
\end{align}

where $p_u$ is the upstream pressure ahead of the bearing entrance and is assumed as the atmospheric pressure in the present work, and $u_{in}$ is the mean velocity of the lubricating flow at the bearing entrance, and $k$ is the loss coefficient. The loss coefficient in the flow model as shown in Fig. 4 includes both effects of the flow loss and the energy supply by the surface friction of the runner. The loss coefficient has been considered as a constant value so far in the performance analysis of the pad bearings. However, since the inlet pressure is strongly affected by the loss coefficient, evaluating the loss coefficient accurately is very important. Figure 5 shows the loss coefficient obtained from the numerical analysis versus the Reynolds number for several cases of the distance between pads. It shows that the loss coefficient can be also negative in the operating conditions with a lower Reynolds number and is significantly varied with the Reynolds number and the distance factor $L_g/h$. A negative loss coefficient means that the energy supply into the flow model shown in Fig. 4 is stronger than the flow loss.

The loss coefficient can be expressed in both terms of the energy supply by the surface friction of the runner and the flow loss owing to the difference in the velocities between the upstream and the bearing entrance. The energy supply effects are inversely proportional to the Reynolds number and the flow loss is closely related to the boundary layer thickness at the upstream ahead of the bearing entrance. The ratio of velocity boundary layer thickness to the lubricating film thickness in the cavity region is proportional to $\mu L_g / (\rho U^2 h^s)$, and is inversely proportional to the flow loss. And the inlet pressure build-up in the pad bearings with a parallel film vanishes when the loss coefficient is 0.75. Therefore, a formula of loss coefficient can be driven as follows using these analytic considerations and the trial and error method:

\begin{align}
k &= - \frac{C_1}{Re} + 0.75(1 - e^{-C_2 \sqrt{Re}/X})
\end{align}

As shown in Table 1, the parameters $C_1$, $C_2$, $n$ have values that change little with the distance factor ($\lambda$). The regressions of these parameters on the distance factor are

\begin{align}
C_1 &= 5.18\lambda^{-0.06}, \quad C_2 = 0.73 \lambda^{-0.05}, \quad n = 1.38 \lambda^{-0.08}
\end{align}

Figure 6 shows the comparisons between the loss coefficients by the present numerical analysis and the results from Eqs. (8) and (9). The two results have good agreement. So, in the future, the inlet pressure build-up can be easily and accurately estimated from Eqs. (8) and (9) instead solving the full numerical analysis. When using these equations for non-parallel bearing film, the Reynolds number and the distance factor should be computed based on the film thickness at the bearing entrance.
Effects of distance Between Pads on Bearing Performance

In order to describe effects of the distance between pads on the bearing performance, film pressure and load capacity are calculated using the formula of Eqs. (8) and (9) for an infinite slider bearing as shown in Fig. 7. Figure 8 shows the film pressure of slider bearing at just only different values of the distance between pads. These results explain that the film pressure is significantly affected by the distance between pads as well as the modified Reynolds number. Figure 9 represents the variations of the load capacity of the slider bearing by the distance between pads versus the modified Reynolds number. The effect of the distance between pads on the load capacity is not much at lower modified Reynolds number but very high at higher modified Reynolds number.

Test Rig

The schematic diagram of the experimental apparatus is shown in Fig. 10. A close-up view of the test pad is shown in Fig. 11. The diameter and the length of the runner (journal) are 196 mm and 104 mm, respectively. The runner is within an oil reservoir made of acryl which is 273 mm in width, 193 mm in length, and 265 mm in height. The geometry and size of the test pad is described in Fig. 12. The machined clearance between the test pad and the runner is 0.15 mm and the material of the pads is duralumin.

The test pad has seven holes (0.6 mm diameter) along the width of the pad for measuring the inlet pressure, at the location of 0.5 mm apart from the leading edge of the pad, and seven holes in the mid plane of the pad for the film pressure as shown in Fig. 12. All the holes of the test pad are connected through the flexible rubber tubes (inner diameter is 1 mm and outer diameter is 2 mm) to the manometers. The flexible tubes do not give any restrictions on the tilt motion of the test pad. The pivot position is the center of the
pad and the pivot film thickness is obtained from averaging the proximity probes mounted at the center of both side of the pad.

The total applied load on the runner is a constant value of 26.5 N in the vertical direction for all the experiments. But the load on the test pad varies with the angle from the vertical axis to the pivot of the test pad as

\[ W = \frac{26.5/2}{\cos(\theta_p + \theta_g/2)} \quad [N], \]  

(10)

where \( \theta_p \) is the angle from the leading edge to the pivot of the test pad, and \( \theta_g \) is the angle between the pads as shown in Fig. 10. The experiments are conducted on the three different angles between the pads, which are 4 deg, 14 deg, and 34 deg corresponding to 6.8 mm, 23.7 mm, and 58.2 mm in the distance between the pads, respectively. Therefore, the applied loads are 14.1 N for \( \theta_g = 4 \) deg, 14.6 N for \( \theta_g = 14 \) deg and 16.2 N for \( \theta_g = 34 \) deg. These are also equivalent to 3.69 kPa, 3.82 kPa and 4.24 kPa mean pressure, which is the load divided by the lubricating surface area of the pad, respectively. The pad preceding the test pad is lubricated by an oil layer on the runner surface which is partially submerged in the lubricating oil. The test pad is designed to be lubricated by an supply of 6.5 liters per minute in the cavity region and the outflow of the previous pad as shown in Fig. 10. The viscosity of lubricant is 4.0 centi-stokes at 40°C and the density is 820 kg/m³. The drain temperature of lubricating oil is about 24°C.

Experimental Results and Comparison With Theoretical Predictions

Procedure for Theoretical Predictions. Theoretical predictions to compare with the experimental results are obtained by controlling the pivot film thickness until the theoretical load is the same as the applied load. The theoretical predictions are obtained using the inlet pressures in the mid plane by the present model with variable loss coefficient by the Eqs. (8) and (9). The inlet pressure profile along the width of the pad is assumed as a parabolic form which is zero at both side corners of the pad and maximum in the mid plane.
Fig. 12  Dimension and geometry of test pad

Fig. 13  Film pressure distribution in the mid plane, $\theta_p = 4$ deg
Comparison With Theoretical Predictions. From the results of numerical analysis, it is found that the inlet pressure build-up is mainly affected by the flow field only corresponding to the boundary layer thickness on the runner surface. That is, the flow conditions far away from the runner surface in the film across are not important in generating the inlet pressure. This means that the inlet pressure build-up expected from the Eqs. \((8)\) and \((9)\) will be reasonable for most actual pad bearings.

Figures 13–15 show the film pressure distributions in the mid plane for different distance between the pads, respectively. The inlet pressure build-up in the mid plane and the ratio of the inlet pressure to the maximum film pressure increase as the rotational speed is higher. In addition, the entire film pressure varies with the distance between two adjacent the pads due to their different applied loads on the test pad. Under the present experimental conditions, the present model, which has the variable loss coefficient by the Eqs. \((8)\) and \((9)\), predicts well the inlet pressure build-up for all the rotational speeds. The experimental conditions look like much different from the numerical solution domain but the inlet pressure build-up can be well estimated by the Eqs. of \((8)\) and \((9)\). This means that the flow conditions far away across from the runner surface do not influence on the inlet pressure build-up.

Figure 16 describes the inlet pressure factor in the mid plane. The experimental results of the inlet pressure factor are much more than unity in lower speeds (lower Reynolds number) and decrease with the rotational speed. The differences in the inlet pressure among the different pad distances increases with the rotational speed. The longer distance has a role to increase the inlet pressure build-up because it provides more time for developing the flow in the cavity region. The present theoretical prediction model gives good agreement with the experimental results at the higher rotational speeds. Now through the present model, the distance between pads can be estimated reasonably from the evaluation of the inlet pressure.

Figure 17 shows the pivot film thickness obtained from the present model and the experiments. The pivot film thickness without the inlet pressure build-up can never be more than the machined clearance, 0.15 mm, theoretically for any operating conditions. But the pivot film thickness increases with the inlet pressure build-up and can be more than the machined clearance. There are some differences in the pivot film thickness because of the different inlet pressure build-up owing to the distance between pads. The overall trend of the theoretical results with the present model shows quite good agreement with the experiments.

Measured inlet pressure distributions are shown in Fig. 18. It is found that the inlet pressure at both side corners of the pad can be more than the atmospheric pressure and the inlet pressure distributions seem to differ from a parabolic form used in the theoretical predictions. The inlet pressure distributions measured may be a polynomial form with the fourth order or more. But using a polynomial expression with a higher order for the inlet pressure distribution deteriorates a convergence of the film pressure in the numerical analysis of the bearing performance, especially at higher Reynolds number. According to our prior research (not described here), the type of function used does not have a significant effect on the bearing performance.

Conclusions

To calculate correctly the inlet pressure build-up, the two-dimensional full Navier-Stokes equations are solved for the cavity
Fig. 15 Film pressure distribution in the mid plane, \( \theta_p = 34 \text{ deg} \)

Fig. 16 Comparison of inlet pressure factor

Fig. 17 Comparison between theoretical and experimental pivot film thickness
region and the lubricating film. The inlet pressure factor can be much more than unity at lower speeds and decreases as the rotational speed (Reynolds number) increase. The distance between pads also has a large influence on the inlet pressure build-up. The most influential parameters on the inlet pressure are the Reynolds number and the distance factor (the distance between pads divided by the inlet film thickness).

The present work proposes a new model able to predict the inlet pressure build-up easily and accurately. The new model can estimate the effects of the distance between the pads and the Reynolds number on the inlet pressure build-up. The theoretical predictions by the new model are in good agreement with the experiments conducted in this study.

Nomenclature

- \( B \) = length of a pad
- \( h \) = film thickness
- \( h_1 \) = inlet film thickness of slider
- \( h_2 \) = outlet film thickness of slider
- \( h^* \) = normalized \( h, h/B \)
- \( H_g \) = height of a cavity between pads
- \( H_g^* \) = normalized \( H_G, H_G/B \)
- \( k \) = loss coefficient
- \( L_d \) = distance between pads
- \( L_d^* \) = normalized \( L_d, L_d/B \)
- \( p \) = film pressure
- \( p_{a} \) = up stream pressure
- \( P^* \) = dimensionless pressure, \( p/p_{U_s}^2 \)
- \( P_i^* \) = inlet pressure factor, \( p_i/p_{U_s}^2 \)
- \( \text{Re} \) = Reynolds number, \( \rho U_s h/\mu \)
- \( \text{Re}^* \) = modified Reynolds number, \( \rho U_s h^2/\mu B \) or \( \rho U_s h^2/\mu B \)
- \( u, v \) = velocity of sliding and film thickness across
- \( U, V \) = dimensionless velocity, \( u/U_s, v/U_s \)
- \( U_s \) = runner speed
- \( W \) = load capacity
- \( W^* \) = dimensionless load capacity, \( Wh^2/\mu U_s B \)

- \( x, y \) = coordinate of sliding and film thickness across
- \( X, Y \) = dimensionless axis, \( x/B, y/B \)
- \( \lambda \) = distance factor, \( L_d/h \)
- \( \theta_0 \) = angle between pads
- \( \mu \) = viscosity of lubricant
- \( \rho \) = density of lubricant

References