

## OPTIMAL DESIGN FOR DYNAMIC SPECTRUM ACCESS IN COGNITIVE RADIO NETWORKS UNDER RAYLEIGH FADING

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**ABSTRACT.** We consider a time slotted cognitive radio network under Rayleigh fading where multiple secondary users (SUs) contend for spectrum usage over available primary users' channels. We analyze the performance of a channel access policy where each SU stochastically determines whether to access a wireless channel or not based on a given access probability. In the analysis, we focus on the queueing performance of an arbitrary SU with the channel access policy. To improve the queueing performance of SUs, the access probability in our channel access policy is adapted to the knowledge on the wireless channel information, e.g., the number of available channels and the nonfading probability of channels. It is then important to obtain the optimal access probabilities from the queueing performance perspective.

In this paper we consider three scenarios. In the first scenario, all SUs have full information on wireless channel status and fading channel conditions. In the second scenario, all SUs have the information on wireless channel status but do not know their fading channel conditions, and in the last scenario all SUs do not have any information on wireless channel status and conditions. For each scenario we analyze the queueing performance of an arbitrary SU and show how to obtain the optimal access probabilities with the help of the effective bandwidth theory. From our analysis we provide an insight on how to design an optimal channel access policy in each scenario. We also show how the optimal channel access policies in three scenarios are related with each other. Numerical results are provided to validate our analysis. In addition, we investigate the performance behaviors of the optimal channel access policies.

**1. Introduction.** With the increased demand for wireless communication, the scarcity in spectrum has become a serious problem, because much of the prime wireless spectrum has been already allocated to licensed users and unlicensed users are not allowed to access them even when they are not used. However, recent studies on the spectrum usage pattern have revealed that the allocated spectrum experiences low utilization [6], [11]. This discrepancy between allocation and usage leads to the introduction of cognitive radio (CR) network.

In CR networks, primary users (PUs) are defined as wireless devices that have the license to access a specific spectrum band. The utilization of the spectrum band can be improved if unlicensed users, usually called secondary users (SUs), are allowed

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to access the spectrum band when it is not used. By exploiting the spectrum, e.g., in an opportunistic fashion, dynamic spectrum access enables SUs to know which portions of the spectrum are available, to select the channel and to coordinate the access to the spectrum. However, PUs have absolute priority to occupy the licensed spectrum in the network, and their communication should not be interfered by SUs. So spectrum sensing by SUs is important in CR networks to explore free spectrum bands and to avoid interference with PUs [4]. To use the channels unoccupied by PUs, called idle channels, SUs first sense wireless channels cooperatively or individually using a given channel sensing policy at each slot. After sensing wireless channels, each SU knows all or partial information on wireless channels regarding whether they are idle or not. Each SU then selects and accesses one or multiple idle channels for wireless communications according to a given channel access policy. Hence, a suitable choice of channel sensing and access policies improves spectral efficiency and network performance.

We assume that all wireless channels of SUs are according to the i.i.d. Rayleigh block fading model where the power gain of each channel remains invariant during a time slot [16, 1]. Note that the fading in wireless channels affects the packet transmissions of SUs. That is, if the power gain of the selected channel by an SU is good enough, the SU can successfully transmit a packet through the channel. Otherwise, the packet transmission by the SU fails.

In this paper, we focus on a channel access policy for SUs under Rayleigh block fading. We assume that all wireless channels are homogeneous. When a channel is occupied by a PU, the channel is called *busy*. Otherwise, the channel is called *idle*. In the channel access policy each SU determines stochastically whether to access a channel at each slot based on *a priori* given access probability (AP). When an SU determines to access a channel, it selects one of idle channels. If two or more SUs select the same idle channel and transmit packets through the channel, there occurs a packet collision. In addition, even when only one SU selects an idle channel but the channel is in a deep fade, the packet transmission by the SU fails. Hence, the AP, controlling the number of SUs who access idle channels, is adapted to the channel status (idle or busy) and conditions (fading or nonfading) in our channel access policy to improve the queueing performance of SUs.

We analyze the queueing performance of an arbitrary SU with the above mentioned channel access policy. To this end, we use the queue length tail probability of an arbitrary SU as our performance metric. To analyze the queueing performance of an arbitrary SU, we consider three scenarios. In the first scenario, all SUs have full information on wireless channel status and conditions. In the second scenario, all SUs have the information on wireless channel status but do not know their channel conditions, and in the last scenario all SUs do not have any information on wireless channel status and conditions.

Noting that the AP significantly affects queueing performance, it is important to obtain the optimal values of APs to optimize queueing performance in each scenario, i.e., to minimize the queue length tail probability, which is one of the main objectives of this study. To obtain the optimal AP values in the channel access policy for each scenario, we first analyze the queueing performance of an arbitrary SU with the help of the effective bandwidth theory. We derive the effective bandwidth function (EBF) of the service capacity process of an arbitrary SU for each scenario. We then analyze the characteristics of the EBF of the service capacity process and

finally obtain the optimal values of APs in each scenario that maximize queueing performance.

The contribution of this paper is as follows. We provide an explicit expression of the optimal AP values in the channel access policy for each scenario, and show the effects of the knowledge on the channel status and conditions on queueing performance. We also show how the optimal channel access policies in three scenarios are related with each other.

The rest of the paper is organized as follows. In section 2 we provide a brief summary on related works. In section 3 we describe the system model. In section 4 the detailed operations of the channel access policy are presented. In section 5 we provide how to obtain the queue length tail probability with the help of the effective bandwidth theory. In section 6 we analyze the performance of the channel access policy in each scenario and obtain the optimal AP values in each scenario. In section 7 we provide numerical results to validate our analysis and to investigate queueing performance. Finally, we provide our conclusions in section 8.

**2. Related work.** Several approaches in a cognitive radio network have been introduced to maximize the total throughput while satisfying the QoS requirements [7, 5, 17, 15]. In [7], Hong et al. considered an optimization problem for both downlink and uplink transmissions to maximize the throughput of a CR network. They proposed two-phase mixed distributed/centralized control algorithms that require minimal cooperation. In [5], Ding et al. proposed a distributed algorithm that jointly addresses routing, dynamic spectrum-assignment, scheduling, and power-allocation for CR ad hoc networks to allocate resources efficiently, distributively, and in a cross-layer fashion. In [17], Stotas and Nallanathan proposed a novel CR system that improves the ergodic throughput by performing data transmission and spectrum sensing at the same time. However, the above mentioned works did not consider queueing performance in CR networks.

Regarding queueing performance, a queueing analytic framework was developed to study the performance of a channel-quality-based opportunistic spectrum access by CR users in a dynamic spectrum access network [14]. They assumed a centralized opportunistic spectrum scheduling for SUs. In [18], Wang et al. focused on the queueing delay performance of SUs in a CR network. They used a fluid queue approximation approach and represented the queue dynamics as Poisson driven stochastic differential equations, but they did not consider the fading effect in wireless channels.

The effective bandwidth theory has been used to analyze the performance of wireless networks [19], [9], [1], [13]. In [1], Akin and Gursoy studied the effective capacity of the cognitive radio channel in order to identify the performance under statistical QoS constraints. They did not consider a channel access policy and the analysis was conducted for several transmission schemes under different assumptions on the availability of the channel information. In [13], Musavian and Aissa derived the optimal rate and power adaptation policy that maximizes the effective capacity of a channel, and provided closed-form expressions for power allocation and effective capacity. In [8], Hwang and Roy considered the optimal channel access policies for CR networks without the fading effect.

### 3. The system model.

**3.1. The cognitive radio network model.** We consider a time slotted cognitive radio (CR) network with  $N$  wireless channels. The time axis is divided into slots of equal size  $T_f$  and time is indexed by  $t$  ( $t = 1, 2, \dots$ ). We assume that there are  $M$  SUs in the CR network.

The wireless channel availability for SUs varies over time depending on the usage pattern of PUs. The wireless channel conditions of SUs also vary over time. Moreover, since multiple SUs contend for channel access at each time slot, we need a channel access policy in the CR network to resolve or alleviate the contention by SUs.

**3.2. Primary user activity.** We assume that the occupancy process of each wireless channel by PUs is modeled by a two state Markov chain with state space  $\{0, 1\}$ . The wireless channel state is defined by 0 if the channel is busy, and by 1 if the channel is idle. We further assume that state transitions occur at slot boundaries and the transition probability matrix of the Markov chain is given by

$$\mathbf{Q} = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}, \quad (1)$$

where  $p$  is the transition probability from state 0 to state 1 and  $q$  is the transition probability from state 1 to state 0.

We assume that the occupancy processes of all wireless channels by PUs are independent. Let  $N(t)$  be the number of idle wireless channels at time slot  $t$ . It is then obvious that  $N(t)$  is a Discrete Time Markov Chain (DTMC) with state space  $\{0, 1, \dots, N\}$ . The transition probability matrix of  $N(t)$  is denoted by  $\mathbf{R} = (R_{kl})$  where  $R_{kl}$  denotes the  $(k, l)$ -th element of matrix  $\mathbf{R}$ . From (1), it is easy to show that  $R_{kl}$  is given by

$$R_{kl} = \sum_{i=\max(0, k+l-N)}^{\min(k, l)} \binom{k}{i} (1-q)^i q^{k-i} \binom{N-k}{l-i} p^{l-i} (1-p)^{N-k-l+i}.$$

**3.3. Wireless channel model.** We adopt that the i.i.d. Rayleigh block fading model for all wireless channels of SUs [16]. For SU  $i$ , its power gain of channel  $j$  at any time slot, denoted by  $f_j^i$ , is then exponentially distributed with mean, say,  $\frac{1}{\mu}$ . Due to the Rayleigh block fading assumption, the power gain  $f_j^i$  of channel  $j$  for SU  $i$  remains invariant during a time slot, but varies across time slots. Furthermore, all power gains  $f_j^i$ ,  $1 \leq i \leq M$ ,  $1 \leq j \leq N$ , are independent.

To capture the fading effect in the packet transmissions of SUs, when an SU, say SU  $i$ , uses an idle wireless channel, say channel  $j$ , we assume that SU  $i$  can successfully transmit a packet through channel  $j$  only if the power gain  $f_j^i$  is above a given threshold, say  $\epsilon$ . Otherwise, we assume the packet transmission by SU  $i$  fails. When  $f_j^i \leq \epsilon$ , the state of channel  $j$  for SU  $i$  is referred to as a deep fade. From our assumption so far, the probability  $s$  that channel  $j$  for SU  $i$  is not in a deep fade is given by

$$s := P\{f_j^i > \epsilon\} = e^{-\mu\epsilon}.$$

From now on,  $s$  is called the nonfading probability.

**4. Wireless channel access policy.** In this section, we focus on a channel access policy by which all SUs contend for channel access. In the channel access policy considered in this paper any SU decides stochastically whether to access a channel at each time slot based on the access probability (AP), independent of all other SUs. When the SU decides to access a channel, it is called an active SU from now on. When two or more active SUs select the same idle channel and transmit their packets, they experience a packet collision. In addition, even when only a single SU selects an idle channel, but the idle channel for the SU is in a deep fade, the SU cannot transmit its packet successfully. Hence, the AP that adapts to the knowledge of the idle channels and their channel conditions, is obviously desirable.

We consider three scenarios and their respective channel access policies in this paper. In the first scenario, called Scenario A, we assume that all SUs have full information on wireless channel status, i.e., which channels are idle and which channels are busy, at each time slot. In addition, we assume that SU  $i$  ( $1 \leq i \leq M$ ) knows all channel conditions, i.e., the power gains  $f_j^i, 1 \leq j \leq N$ , at each time slot. In this scenario, when  $N(t) = k, 0 \leq k \leq N$ , each SU determines to access an idle channel (becomes active) with probability  $a_k$ , and does not access any idle channel (becomes inactive) with probability  $1 - a_k$ . Then, any active SU randomly selects one channel among the idle channels which are nonfading for the SU. If there are no available channels (i.e., no idle channels or all idle channels are in deep fades) for the active SU, the SU does not select any channel. Hence, the packet transmission of the active SU is successful if no other active SUs select the same idle channel for packet transmission.

In the second scenario, called Scenario B, we assume that all SUs have the information on wireless channel status at each time slot as in Scenario A. However, all SUs do not know their channel conditions at the beginning of each time slot. In this scenario, as in Scenario A, when  $N(t) = k, 0 \leq k \leq N$ , each SU determines to access an idle channel with probability  $b_k$ , and does not access any idle channel with probability  $1 - b_k$ . Then, any active SU randomly selects one of idle channels and if the selected channel is not in a deep fade after checking the power gain of the selected channel, the active SU transmits a packet. Hence, contrary to Scenario A, the packet transmission of the active SU is successful in this scenario if the selected channel for the active SU is not in a deep fade and any other active SUs do not select the same channel or select the same channel but the channel for them is in a deep fade.

In the third scenario, called Scenario C, we assume that all SUs do not have any information on wireless channel status and conditions at the beginning of each time slot. In this scenario, the AP cannot be adapted to the wireless channel status, and hence each SU determines to access a channel with a unified probability  $c$ , and does not access any channel with probability  $1 - c$ . Then, any active SU randomly selects one channel among  $N$  channels and if the selected channel is not in a deep fade after checking the power gain of the selected channel, the active SU transmits a packet. So the packet transmission of the active SU is successful in this scenario if the selected channel is neither busy nor in a deep fade and any other active SUs do not select the same channel or select the same channel but the channel for them is in a deep fade.

For each scenario, we will analyze the channel access policy from the queueing performance perspective and obtain the optimal APs that optimize the queueing performance. In addition, we investigate the effect of knowledge of channel status

(busy and idle) and conditions (fading and nonfading) on the performance of the channel access policy by comparing the results of three scenarios.

**5. Queueing performance and effective bandwidth theory.** We assume that each SU has a buffer to accommodate its packets at the MAC layer. We consider an arbitrary SU as our reference and call it the tagged SU. We analyze the queueing performance of the tagged SU under the three scenarios explained in section 4.

To analyze the queueing performance of the tagged SU, we assume that all the other SUs always have packets to transmit. This is the worst case analysis for the tagged SU. Now consider the service capacity process of the tagged SU at the MAC layer. Let  $\{c_k(t)\}$  be the service capacity of the tagged SU. That is, assuming that the tagged SU has a packet to transmit, when  $N(t) = k$  at time slot  $t$ ,  $c_k(t)$  is defined by 1 if the packet transmission of the tagged SU is successful at time slot  $t$  and by 0 otherwise. Then  $c_{N(t)}(t)$  denotes the service capacity process of the tagged SU. Note that  $\{c_k(t)\}$  is not the actual packet service process.

Let  $q(t)$  denote the queue length (i.e., the number of packets in the queue) of the tagged SU at time slot  $t$ . Let  $a(t)$  denote the number of packets newly generated at the tagged SU during time slot  $t$ . Since  $c_{N(t)}(t)$  denotes the number of successfully transmitted packets at time slot  $t$ , the queueing process  $\{q(t)\}$  of the tagged SU evolves as follows [2], [3]:

$$q(0) = 0, \quad q(t+1) = \max\{0, q(t) + a(t) - c_{N(t)}(t)\}, \quad t \geq 0,$$

where  $\max\{x, y\}$  denotes the maximum of  $x$  and  $y$ .

To analyze the queueing process  $\{q(t)\}$ , we use the effective bandwidth theory in this paper. In the effective bandwidth theory, we need to compute two effective bandwidth functions (EBFs): one for the service capacity process  $c_{N(t)}(t)$  and the other for the packet arrival process  $a(t)$ .

We start with deriving the EBF for the service capacity process for the tagged SU. Let  $C(t)$  denote the cumulative service capacity process during the interval  $[0, t)$ , i.e.,  $C(t) = \sum_{s=0}^{t-1} c_{N(s)}(s)$ . Let  $\Lambda_C(\theta)$  denote the Gärtner-Ellis limit of the cumulative service capacity process  $C(t)$ , i.e.,  $\Lambda_C(\theta) = \lim_{t \rightarrow \infty} \frac{1}{t} \log(E[\exp(\theta C(t))])$ , provided that the limit exists. Then the Effective Bandwidth Function (EBF) of the packet service process of the tagged SU is defined by [2], [19]

$$\xi_C(\theta) := -\frac{\Lambda_C(-\theta)}{\theta}.$$

The EBF of the service capacity process,  $\xi_C(\theta)$  captures the stochastic properties of the service capacity process  $c_{N(t)}(t)$ . For instance, when  $\theta \rightarrow 0$ , it converges to the average service rate. In contrast, when  $\theta \rightarrow \infty$ , it converges to the minimum service rate [2],[20]. To compute the EBF of the service capacity process, let  $\Phi(\theta)$  be the diagonal matrix with diagonal elements  $\{\phi_0(\theta), \phi_1(\theta), \dots, \phi_N(\theta)\}$ , where  $\phi_k(\theta)$  are defined by

$$\phi_k(\theta) := E[e^{\theta c_{N(t)}(t)} | N(t) = k], \quad 0 \leq k \leq N. \quad (2)$$

Since the service capacity process  $\{c_{N(t)}(t)\}$  is a Markov modulated process, it can be shown that the EBF of the packet service process is given by

$$\xi_C(\theta) = -\frac{\log \delta_C(-\theta)}{\theta},$$

where  $\delta_C(\theta)$  is the Perron-Frobenius (PF) eigenvalue of  $\Phi(\theta)R$  and  $R$  is given in Section 3.2. The proof can be found in [2],[3].

We next derive the EBF of the packet arrival process at the tagged SU. Let  $A(t)$  denote the cumulative arrival process during the interval  $[0, t)$ , i.e.,  $A(t) = \sum_{n=0}^{t-1} a(n)$ . We define the EBF of the arrival process,  $\xi_A(\theta)$ , by [2], [19]

$$\xi_A(\theta) := \frac{\Lambda_A(\theta)}{\theta},$$

where the Gärtner-Ellis limit of the cumulative arrival process is defined by

$$\Lambda_A(\theta) = \lim_{t \rightarrow \infty} \frac{1}{t} \log(E[\exp(\theta A(t))]),$$

provided that the limit exists. For instance, when  $\theta \rightarrow 0$ , it converges to the average arrival rate; for  $\theta \rightarrow \infty$ , it converges to the peak arrival rate. For later use, we provide an explicit expression for the EBF of the packet arrival process,  $\xi_A(\theta)$ . For simplicity, we assume that the packet arrival process at the tagged SU is according to a Poisson process with arrival rate  $\lambda$  (packets/slot). In this case, the EBF of the packet arrival process is given by [2]

$$\xi_A(\theta) = \frac{\lambda(e^\theta - 1)}{\theta}.$$

Even though we assume the Poisson arrivals in this study, our analytic framework can accommodate more general stationary arrival processes of which Gärtner-Ellis limits exist and are differentiable for all  $\theta$  [2], e.g., Markov Modulated Poisson Process (MMPP).

We are now ready to investigate the queueing performance with the help of the EBFs of the service capacity and arrival processes. Let  $q(\infty)$  denote the random variable representing the queue length in the steady state. It is known that under the stable condition, the tail probability  $P(q(\infty) > x)$  in the steady state is approximately given by [2],[19],[10]

$$P(q(\infty) > x) \approx P(q(\infty) > 0) \exp(-\theta^* x), \quad (3)$$

where  $\theta^*$  is the unique solution of the equation

$$\xi_A(\theta) = \xi_C(\theta). \quad (4)$$

In addition, we have

$$P(q(\infty) > 0) = \frac{\xi_A(0)}{\xi_C(0)}. \quad (5)$$

Recall that one of our objectives is to determine the optimal values of APs for each scenario to optimize the queueing performance of the tagged SU, i.e., to minimize the queue length tail probability given in (3). We note that the values of APs obviously affect the service capacity process  $C_{N(t)}(t)$  and, in turn, the EBF of the service capacity process of the tagged SU. So for each scenario, if the EBF of the service capacity process for each  $\theta$  is increased by the change of the APs, then the solution  $\theta^*$  of (4) increases and the probability in (5) decreases. Consequently, from (3) we see that the tail probability decreases. This observation shows that, if we obtain the values of APs to which the corresponding EBF of the service capacity process is the largest, then they are, in fact, the optimal values of APs that maximize the solution  $\theta^*$  and  $\xi_C(0)$ , and hence minimize the queue length tail probability.

**6. Performance analysis.** In this section, we analyze the characteristics of the EBF of the service capacity process and show how to obtain the optimal values of APs for each scenario.

Given that the nonfading probability is  $s$ , let  $E_j^i(s)$  be the probability that there are  $j$  SUs among the total of  $i$  SUs for which a wireless channel is nonfading, and  $F_j^i(s)$  be the probability that there are  $j$  nonfading channels among the total of  $i$  channels for an SU. Since all channels of SUs are i.i.d, the probabilities  $E_j^i(s)$  and  $F_j^i(s)$  are given by

$$\begin{aligned} E_j^i(s) &= \binom{i}{j} s^j (1-s)^{i-j}, \\ F_j^i(s) &= \binom{i}{j} s^j (1-s)^{i-j}. \end{aligned}$$

Since each SU independently determines whether to access the channels with the same AP, the probability  $P_j^i(x)$  that there are  $j$  active SUs among the total of  $i$  SUs when the AP is  $x$ , is given by

$$P_j^i(x) = \binom{i}{j} x^j (1-x)^{i-j}.$$

**6.1. Scenario A.** In this subsection, we consider that all SUs have full information on the channel status (busy or idle) and the channel conditions (fading or nonfading). We start with the derivation of  $\phi_k(\theta)$  in (2). By abuse of notation, we use  $\phi_k^A(\theta)$  for Scenario A instead of  $\phi_k(\theta)$ .

When  $k=0$ , it is obvious that  $\phi_0^A(\theta) = 1$  because  $C_{N(t)}(t) = 0$  for  $N(t) = 0$ . For  $1 \leq k \leq N$ ,  $\phi_k^A(\theta)$  is given as in (6).

$$\begin{aligned} \phi_k^A(\theta) &:= E[e^{\theta c_{N(t)}(t)} | N(t) = k] \\ &= a_k (1 - (1-s)^k) \sum_{i=0}^{M-1} E_i^{M-1}(s) \left\{ \sum_{j=0}^i P_j^i(a_k) \times \left( \sum_{h=1}^{k-1} F_h^{k-1}(s) \frac{h}{h+1} \right)^j \right\} e^\theta \\ &\quad + 1 - a_k (1 - (1-s)^k) \sum_{i=0}^{M-1} E_i^{M-1}(s) \left\{ \sum_{j=0}^i P_j^i(a_k) \times \left( \sum_{h=1}^{k-1} F_h^{k-1}(s) \frac{h}{h+1} \right)^j \right\} \\ &= a_k (1 - (1-s)^k) \sum_{i=0}^{M-1} E_i^{M-1}(s) \left\{ \sum_{j=0}^i P_j^i(a_k) \times \left( 1 - \frac{1 - (1-s)^k}{ks} \right)^j \right\} e^\theta \\ &\quad + 1 - a_k (1 - (1-s)^k) \sum_{i=0}^{M-1} E_i^{M-1}(s) \left\{ \sum_{j=0}^i P_j^i(a_k) \times \left( 1 - \frac{1 - (1-s)^k}{ks} \right)^j \right\} \\ &= a_k (1 - (1-s)^k) \sum_{i=0}^{M-1} E_i^{M-1}(s) \left( a_k - \frac{1 - (1-s)^k}{ks} a_k + 1 - a_k \right)^i e^\theta \\ &\quad + 1 - a_k (1 - (1-s)^k) \sum_{i=0}^{M-1} E_i^{M-1}(s) \left( a_k - \frac{1 - (1-s)^k}{ks} a_k + 1 - a_k \right)^i \\ &= 1 + a_k (1 - (1-s)^k) \left( s - \frac{1 - (1-s)^k}{k} a_k + 1 - s \right)^{M-1} (e^\theta - 1) \\ &= 1 + a_k (1 - (1-s)^k) \left( 1 - \frac{1 - (1-s)^k}{k} a_k \right)^{M-1} (e^\theta - 1). \end{aligned} \tag{6}$$

In the right hand side (RHS) of the second equation in (6),  $a_k$  is the probability that the tagged SU becomes active,  $(1 - (1-s)^k)$  is the probability that there is at



least one nonfading channel for the tagged SU among  $k$  idle channels,  $E_i^{M-1}(s)$  is the probability that there are  $i$  untagged SUs for which the selected channel by the tagged SU is also nonfading (they are called untagged SUs of interest),  $P_j^i(a_k)$  is the probability that  $j$  untagged SUs of interest become active among the  $i$  untagged SUs of interest, and  $\left(\sum_{h=1}^{k-1} F_h^{k-1} \frac{h}{h+1}\right)^j$  is the probability that all  $j$  untagged active SUs of interest do not select the same channel that the tagged SU selects.

Our next step to obtain the optimal APs  $\{a_1^*, a_2^*, \dots, a_N^*\}$  that maximize the solution  $\theta^*$  of (4), i.e.,  $\{a_1^*, a_2^*, \dots, a_N^*\}$  optimize the queueing performance of the tagged SU. With the help of the effective bandwidth theory, we can prove the following main theorem.

**Theorem 6.1.** *The optimal values of APs  $\{a_1^*, a_2^*, \dots, a_N^*\}$  are given by*

$$a_k^* = \min\left(\frac{k}{M(1 - (1-s)^k)}, 1\right).$$

For the proof of Theorem 6.1, see Appendix.

Note that  $M(1 - (1-s)^k)$  is the number of SUs that have at least one nonfading idle channel. So Theorem 6.1 reveals that the optimal APs  $a_k^*$ ,  $k = 1 \dots N$  in Scenario A, are proportional to the number of idle channels ( $k$ ) but are inversely proportional to the number of SUs having at least one nonfading idle channel ( $M(1 - (1-s)^k)$ ).

**6.2. Scenario B.** In this subsection we consider that all SU have the information on wireless channel status (busy or idle), but do not know their channel conditions (fading and nonfading) at the beginning of a slot. In this scenario, as in Scenario A, when  $N(t) = k$ ,  $0 \leq k \leq N$ , each SU determines to access an idle channel with probability  $b_k$ , and does not access any idle channel with probability  $1 - b_k$ . Then, any active SU randomly selects one of idle channels and if the selected channel is not in a deep fade after checking the power gain of the selected channel, the active SU transmits a packet. So the packet transmission of the tagged SU is successful if the selected channel by the tagged SU is not in a deep fade and any other active SUs do not select the same channel or select the same channel but the channel for them is in a deep fade.

To get the optimal values  $\{b_1^*, b_2^*, \dots, b_N^*\}$  of APs in this scenario, we first calculate  $\phi_k(\theta)$  defined in (2). By abuse of notation as in Section 6.1, we use  $\phi_k^B(\theta)$  instead of  $\phi_k(\theta)$  in this scenario. When  $k = 0$ , we obviously have  $\phi_0^B(\theta) = 1$ . For  $1 \leq k \leq N$ ,  $\phi_k^B(\theta)$  is given as in (7).

$$\begin{aligned} \phi_k^B(\theta) &:= E[e^{\theta c_{N(t)}(t)} | N(t) = k] \\ &= b_k s \sum_{i=0}^{M-1} P_i^{M-1}(b_k) \sum_{j=0}^i \binom{i}{j} \left(\frac{1}{k}\right)^j \left(\frac{k-1}{k}\right)^{i-j} (1-s)^j e^\theta \\ &\quad + 1 - b_k s \sum_{i=0}^{M-1} P_i^{M-1}(b_k) \sum_{j=0}^i \binom{i}{j} \left(\frac{1}{k}\right)^j \left(\frac{k-1}{k}\right)^{i-j} (1-s)^j \\ &= b_k s \sum_{i=0}^{M-1} P_i^{M-1}(b_k) \left(1 - \frac{s}{k}\right)^i e^\theta + 1 - b_k s \sum_{i=0}^{M-1} P_i^{M-1}(b_k) \left(1 - \frac{s}{k}\right)^i \\ &= b_k s \left(1 - \frac{s}{k} b_k\right)^{M-1} e^\theta + 1 - b_k s \left(1 - \frac{s}{k} b_k\right)^{M-1} \\ &= 1 + b_k s \left(1 - \frac{s}{k} b_k\right)^{M-1} (e^\theta - 1). \end{aligned} \tag{7}$$

In the RHS of the second equation in (7),  $b_k$  is the probability that the tagged SU becomes active,  $s$  is the probability that the selected channel is not in a deep fade,  $P_i^{M-1}(b_k)$  is the probability that there are  $i$  untagged active SUs in the network, and  $\binom{i}{j}(\frac{1}{k})^j(\frac{k-1}{k})^{i-j}(1-s)^j$  is the probability that  $j$  untagged active SUs select the same channel that the tagged SU selects but the channel is in a deep fade for all of the  $j$  untagged active SUs.

Using  $\phi_k^B(\theta)$  in (7) and a similar argument as in the proof of Theorem 6.1, we obtain the optimal values of APs in this scenario as given in the following theorem.

**Theorem 6.2.** *The optimal values of APs  $\{b_1^*, b_2^*, \dots, b_N^*\}$  in Scenario B are given by*

$$b_k^* = \min\left(\frac{k}{sM}, 1\right).$$

Note that  $sM$  is the average number of SUs having good channel condition for an arbitrary wireless channel. So we see from Theorem 6.2 that the optimal AP,  $b_k^*$ ,  $k = 1 \dots N$ , in Scenario B is proportional to the number of idle channels ( $k$ ) and inversely proportional to the average number of SUs having good channel condition for an arbitrary wireless channel ( $sM$ ).

**6.3. Scenario C.** In this subsection we consider Scenario C where all SUs have no information on wireless channels, i.e., no information on channel status and conditions. In this case, any SU cannot adapt the AP according to the wireless channel status and conditions, and hence must use a unified AP,  $c$ , to determine whether they are active or not to access channels. Then, when any SU becomes active with probability  $c$ , the active SU randomly selects one of  $N$  channels and check  $s$  the power gain of the selected channel. If the selected channel is not in a deep fade, the active SU transmits a packet. So the packet transmission of the tagged SU is successful in this scenario if the selected channel is neither busy nor in a deep fade and any other active SUs do not select the same channel or select the same channel but the channel for them is in a deep fade.

To get the optimal value AP  $c^*$ , we start with  $\phi_k(\theta)$ , but we use  $\phi_k^C(\theta)$  in this scenario instead of  $\phi_k(\theta)$ .

When  $k = 0$ , we obviously have  $\phi_0^C(\theta) = 1$ . For  $1 \leq k \leq N$ ,  $\phi_k^C(\theta)$  is given as in (8).

$$\begin{aligned} \phi_k^C(\theta) &:= E[e^{\theta c_{N(t)}(t)} | N(t) = k] \\ &= c \frac{k}{N} s \sum_{i=0}^{M-1} P_i^{M-1}(c) \sum_{j=0}^i \binom{i}{j} \left(\frac{1}{N}\right)^j \left(\frac{N-1}{N}\right)^{i-j} (1-s)^j e^\theta \\ &\quad + 1 - c \frac{k}{N} s \sum_{i=0}^{M-1} P_i^{M-1}(c) \sum_{j=0}^i \binom{i}{j} \left(\frac{1}{N}\right)^j \left(\frac{N-1}{N}\right)^{i-j} (1-s)^j e^\theta \\ &= c \frac{k}{N} s \sum_{i=0}^{M-1} P_i^{M-1}(c) \left(1 - \frac{s}{N}\right)^i e^\theta + 1 - c \frac{k}{N} s \sum_{i=0}^{M-1} P_i^{M-1}(c) \left(1 - \frac{s}{N}\right)^i \\ &= 1 + c \frac{k}{N} s \left(a - \frac{s}{N}c + 1 - c\right)^{M-1} (e^\theta - 1) \\ &= 1 + c \frac{k}{N} s \left(1 - \frac{s}{N}c\right)^{M-1} (e^\theta - 1). \end{aligned} \tag{8}$$

In the RHS of the second equation in (8),  $c$  is the probability that the tagged SU becomes active,  $\frac{k}{N}$  is the probability that the tagged SU selects an idle channel,  $s$  is the probability that the idle channel selected by the tagged SU is not in a deep fade,  $P_i^{M-1}(c)$  is the probability that there are  $i$  untagged active SUs, and  $\binom{i}{j}(\frac{1}{N})^j(\frac{N-1}{N})^{i-j}(1-s)^j$  is the probability that  $j$  untagged active SUs select the same channel that the tagged SU selects but the channel is in a deep fade for all of  $j$  untagged active SUs.

Using  $\phi_k^C(\theta)$  in (8) and a similar argument as in the proof of Theorem 6.1, we obtain the following theorem.

**Theorem 6.3.** *The optimal value  $c^*$  of the AP in Scenario C is given by*

$$c^* = \min\left(\frac{N}{sM}, 1\right).$$

By comparing Theorem 6.2 and Theorem 6.3, the difference in the optimal APs between Scenario B and Scenario C is the numerator. Recalling that the numerator in the optimal APs in Scenario B indicates the number of idle channels, Theorem 6.3 implies that the optimal queueing performance in Scenario C is achieved when all SUs always behave as if all channels were idle. A similar result is shown in [8] where no fading effect of the channels is considered.

From Theorems 6.1, 6.2 and 6.3, we can obtain an interesting result on the relationship between the performances of the optimal channel access policies for the three scenarios, which is summarized in the following proposition.

**Proposition 1.** *Suppose that  $N \leq sM$ . Then the optimal channel access policies for the three scenarios result in the same queueing performance.*

*Proof.* To prove our proposition, we consider  $\phi_k^A(\theta)$  in (6),  $\phi_k^B(\theta)$  in (7) and  $\phi_k^C(\theta)$  in (8). When  $k = 0$ ,

$$\phi_0^A(\theta) = \phi_0^B(\theta) = \phi_0^C(\theta) = 1.$$

For  $1 \leq k \leq N$ , since  $N \leq sM$ , the inequalities,  $k \leq (1 - (1-s)^k)M$  and  $k \leq sM$ , are both satisfied. So the optimal APs in Scenario A are given by  $a_k^* = \frac{k}{M(1-(1-s)^k)}$ . By substituting  $a_k^*$  for  $a_k$  in  $\phi_k^A(\theta)$ , we get from (6) that

$$\phi_k^A(\theta)|_{a_k=a_k^*} = 1 + \frac{k}{M}\left(1 - \frac{1}{M}\right)^{M-1}(e^\theta - 1).$$

Since the optimal APs in Scenario B are give by  $b_k^* = \frac{k}{sM}$ , we obtain from (7) that

$$\phi_k^B(\theta)|_{b_k=b_k^*} = 1 + \frac{k}{M}\left(1 - \frac{1}{M}\right)^{M-1}(e^\theta - 1).$$

Similarly, since the optimal AP in Scenario C is give by  $c^* = \frac{N}{sM}$ , we get from (8) that

$$\phi_k^C(\theta)|_{c=c^*} = 1 + \frac{k}{M}\left(1 - \frac{1}{M}\right)^{M-1}(e^\theta - 1).$$

Therefore

$$\phi_k^A(\theta) = \phi_k^B(\theta) = \phi_k^C(\theta).$$

The respective elements of the matrix  $\Phi(\theta)R$  defined in Section 3 for three scenarios are all identical, and accordingly the resulting PF eigenvalues are the same, i.e., the EBFs of the service capacity processes are identical for three scenarios. This shows the equivalence relationship on the queueing performances of the optimal access policies for three scenarios.  $\square$

**7. Numerical and simulation results.** In this section we provide numerical results to investigate the queueing performance of our channel access policy. To this end, we consider a CR network with 5 wireless channels and 8 SUs. The state transition probabilities of a wireless channel are given by  $p = 0.75$  and  $q = 0.35$ . Then the stationary probability that a channel is busy (idle) is given by  $\frac{q}{p+q} = 0.32$  ( $\frac{p}{p+q} = 0.68$ ), i.e., the average occupancy of a channel by PUs is 32%. The probability that the channel is not in a deep fade is given by  $s = 0.7$ . This corresponds to the threshold  $\epsilon = 0.7133$  when the average power gain of the channel is 2. The arrival process is assumed to be the Poisson process with arrival rate 0.12 (packets/slot).

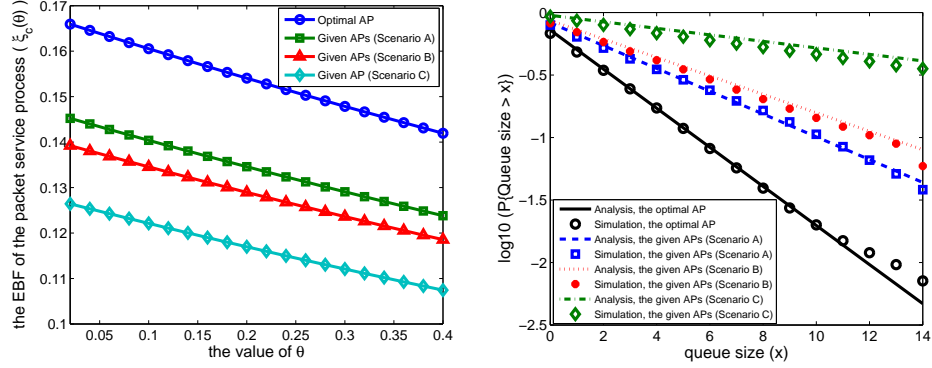
**7.1. Simulation methodology.** We simulate the cognitive radio network using Matlab. Each simulation run is performed for  $3 \times 10^6$  time slots. In each time slot the channel states of all users are changed based on given transition probabilities and the fading channel conditions of all users are according to the Rayleigh block fading model. The queue length is stored at every time slot to get the time average queue length distribution.

**7.2. Impact of the access probabilities on queueing performance.** We assume that the APs  $\{a_1, a_2, \dots, a_5\}$  in Scenario A and  $\{b_1, b_2, \dots, b_5\}$  in Scenario B are both given by  $\{0.8, 0.75, 0.5, 0.45, 0.3\}$  and the unified AP  $c$  in Scenario C is given by 0.4. With the given APs and unified AP the resulting EBFs of the service capacity processes are plotted in Fig. 1(a). As shown in the figure, the EBF of the service capacity process increases for each  $\theta$  when we have more information on the wireless channels provided that the given APs are not optimal. That is, when the same APs are used in Scenario A and Scenario B, the resulting EBF of the service capacity process in Scenario A is larger than that in the Scenario B for each  $\theta$ . This implies that the acquisition of more knowledge on the wireless channels can improve the queueing performance, in general.

To validate our analytic results, we also plot in Fig. 1(a) the EBF of the service capacity process when we use the optimal APs obtained by our analysis. From Proposition 1, we know the optimal channel access policies of three scenarios are all equivalent when  $N \leq sM$ . So we only plot the EBF of the service capacity process in Scenario A with its optimal APs  $\{a_1^*, \dots, a_5^*\}$ . As shown in the figure, the use of the optimal APs in all scenarios results in the largest EBF of the service capacity process. This validates our analysis.

To see the corresponding queueing performance, we plot in Fig. 1(b) the tail probability  $P\{q(\infty) > x\}$  when we use the same APs and the unified AP as given in Fig. 1(a). We also plot in Fig. 1(b) the tail probability when we use the optimal APs. For comparison purpose, we provide the simulation results in the figure. From now on, the tail probabilities obtained from our analysis are denoted by ‘‘Analysis’’ in the figure and those obtained from simulation are denoted by ‘‘Simulation’’ in the figure. As seen in the figure, analytic and simulation results match very well in all cases. The queueing performance in Scenario A with the given APs is better than that in Scenario B with the same APs. In addition, we see in the figure that the use of the optimal APs results in the optimal queueing performance, which validates our analysis.

From our analysis we can compute the average queueing delay of the tagged SU approximately, because the effective bandwidth theory gives us the asymptotic behavior of the tail probability. Using the Little’s Formula, the average queueing



(a) The effective bandwidth function vs.  $\theta$  (b) The queuing performance of channel access policies

FIGURE 1. Impact of access probabilities on queuing performance

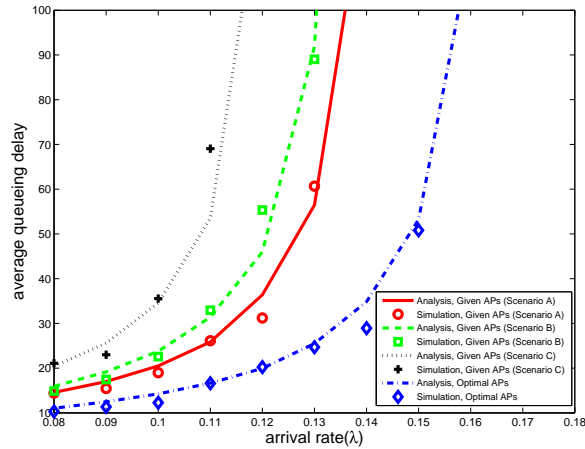


FIGURE 2. The average queuing delay

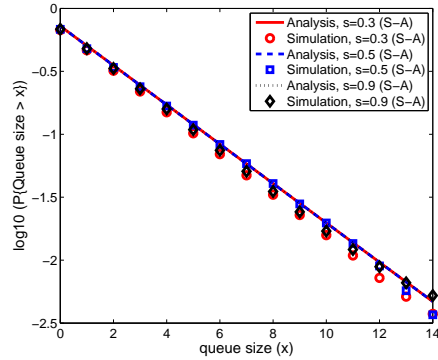
delay of the tagged SU,  $E[D]$ , is given by

$$E[D] = \frac{\sum_{k=0}^{\infty} P\{q(\infty) > k\}}{\lambda} \approx \frac{\sum_{k=0}^{\infty} P\{q(\infty) > 0\} \exp(-\theta^* k)}{\lambda}.$$

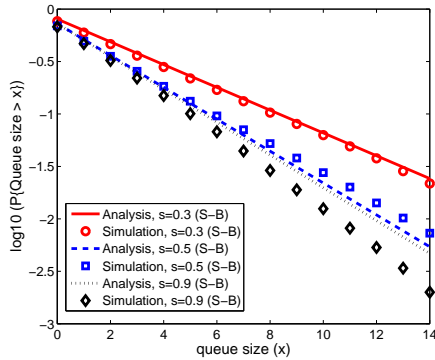
Using the above approximation, we plot in Fig. 2 the average queuing delays in three scenarios when we change the arrival rate  $\lambda$  from 0.08 to 0.18. We also plot the simulation results in the figure.

As clearly seen from Fig. 2, analytic and simulation results on the average queuing delay match very well. In addition, we see that, when we use the optimal APs, the average queuing delay is the lowest in all scenarios.

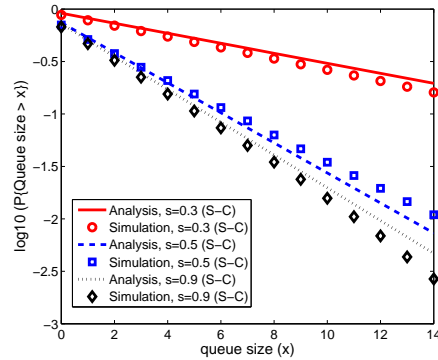
**7.3. Effect of the nonfading probability on queuing performance.** We investigate the effect of the nonfading probability  $s$  (the fading effect) on queuing



(a) Scenario A



(b) Scenario B



(c) Scenario C

FIGURE 3. The queueing performance vs. the nonfading probability ( $s$ )

performance of the optimal channel access policies in three scenarios. For this purpose, we change the value of  $s$  from 0.3 to 0.9 when we use the same parameter values for the CR network as given in Fig. 1 and the optimal APs are used. The resulting tail probabilities are plotted in Fig. 3. As seen in the figure, the difference in the tail probability becomes more significant as we go from Scenario A to Scenario C. Note here that the resulting tail probabilities in three scenarios are not always the same, even though we use the optimal APs. This is because the condition  $N \leq sM$  for the equivalence in Proposition 1 is not always satisfied when we change the value of  $s$  from 0.3 to 0.9. The result implies that the knowledge on the channel status and conditions will be greatly helpful to improve the queueing performance especially when the nonfading probability  $s$  is small. We also see that the queueing performance in Scenario A is not affected by the change of the nonfading probability  $s$  in this example. This implies that, when we know the perfect knowledge on the wireless channels (i.e., the channel status and conditions) as in Scenario A, the use of the optimal APs in the channel access policy can remove the effect of the nonfading probability  $s$  (the fading effect) on queueing performance up to a certain point, which is one of the benefits of the channel access policy in Scenario A.

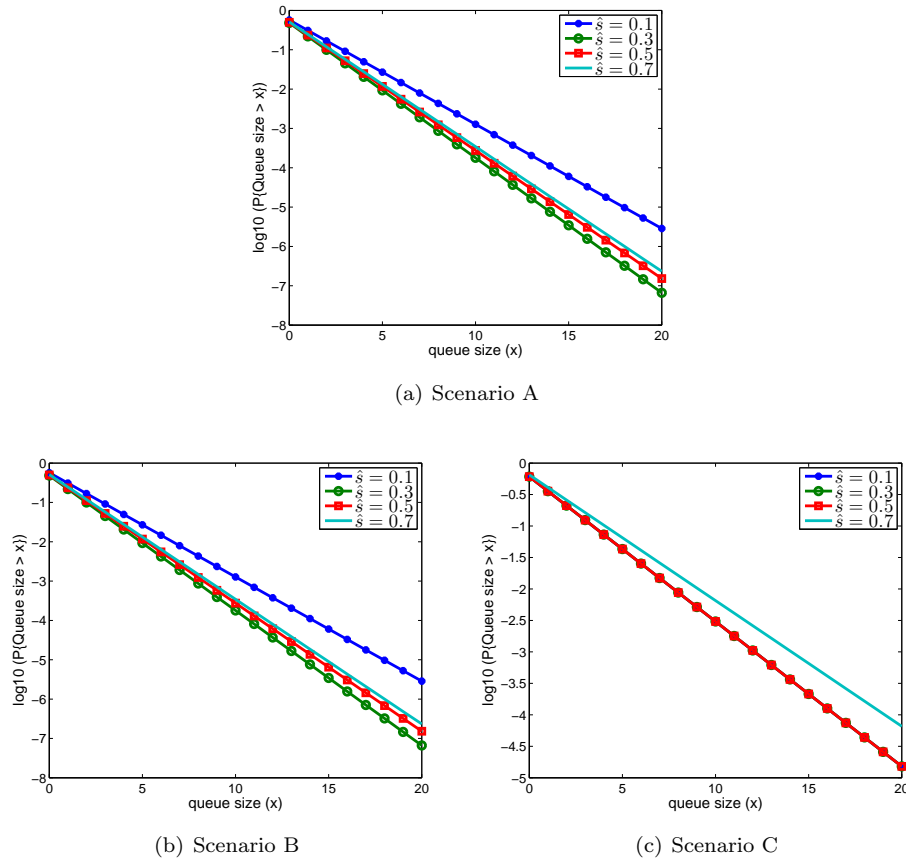
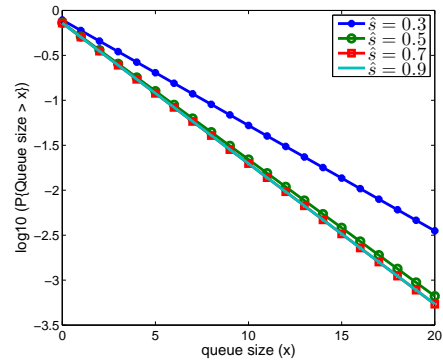


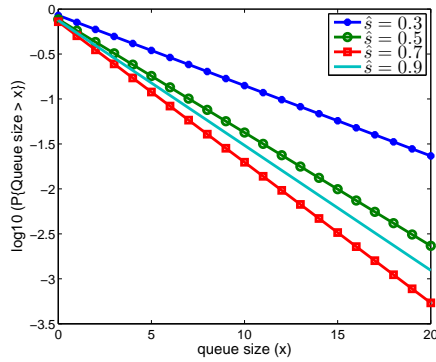
FIGURE 4. The sensitivity of estimating the nonfading probability ( $s = 0.3$ )

In the optimal design of the channel access policy for a CR network, our analysis shows that the knowledge of the channel status and conditions is important. However, while the SUs may obtain reliable channel status information via a centralized database mandated by the FCC [6], it is not easy for the SUs to obtain the channel conditions at each time slot because the channel conditions vary in time and the nonfading probability  $s$  can be obtained only through a measurement of the channel condition in practice. For this reason we need to analyze the sensitivity of the nonfading probability  $s$  to queueing performance. That is, we assume that the nonfading probability  $s$  is estimated through a measurement of the channel condition. The estimated nonfading probability is denoted by  $\hat{s}$ . Note that the estimated probability  $\hat{s}$  is not always equal to the true nonfading probability  $s$ , in general. We then use the estimated value  $\hat{s}$  to obtain the optimal APs and obtain the resulting queueing performance in each scenario.

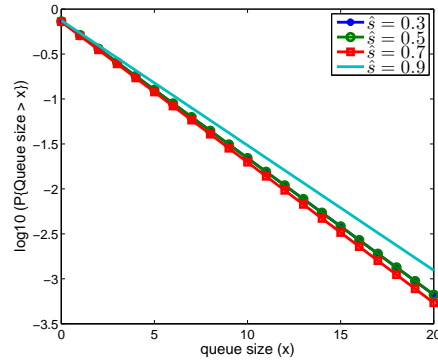
In our numerical analysis, we consider two cases where the true probability  $s$  is set to either  $s = 0.3$  (channel conditions are relatively bad) or  $s = 0.7$  (channel conditions are relatively good). In the first case of  $s = 0.3$ , we change the estimated probability  $\hat{s}$  from 0.1 to 0.7 when the Poisson arrival rate is set to 0.08. We plot



(a) Scenario A



(b) Scenario B



(c) Scenario C

FIGURE 5. The sensitivity of estimating the nonfading probability ( $s = 0.7$ )

the resulting tail probabilities in Fig. 4. In the second case of  $s = 0.7$ , we change the estimated probability  $\hat{s}$  from 0.3 to 0.9 when the Poisson arrival rate is set to 0.12. We plot the resulting tail probabilities in Fig. 5. We omit the simulation results in Figs. 4,5 for better presentations. Obviously, we can expect that the queueing performance degrades when there is an error in the estimation of the nonfading probability  $s$ . This negative effect is shown well in both Fig. 4 and Fig. 5. However, we also see the following interesting observations for three scenarios. In Scenario A, from Fig. 4(a) and Fig. 5(a) we see that queueing performance degradation occurs due to the estimation error of the probability  $s$  and becomes more significant when the wireless channel condition is relatively bad (i.e.,  $s = 0.3$ ). In Scenario B, from Fig. 4(b) and Fig. 5(b), we also see that queueing performance degradation occurs due to the estimation error of the probability  $s$ . However, in contrast to Scenario A, queueing performance degradation becomes more significant when the wireless channel condition is relatively good. Finally, in Scenario C, from Fig. 4(c) and Fig. 5(c) we see that the effect of the estimation error of the nonfading probability  $s$  is relatively less significant on queueing performance.



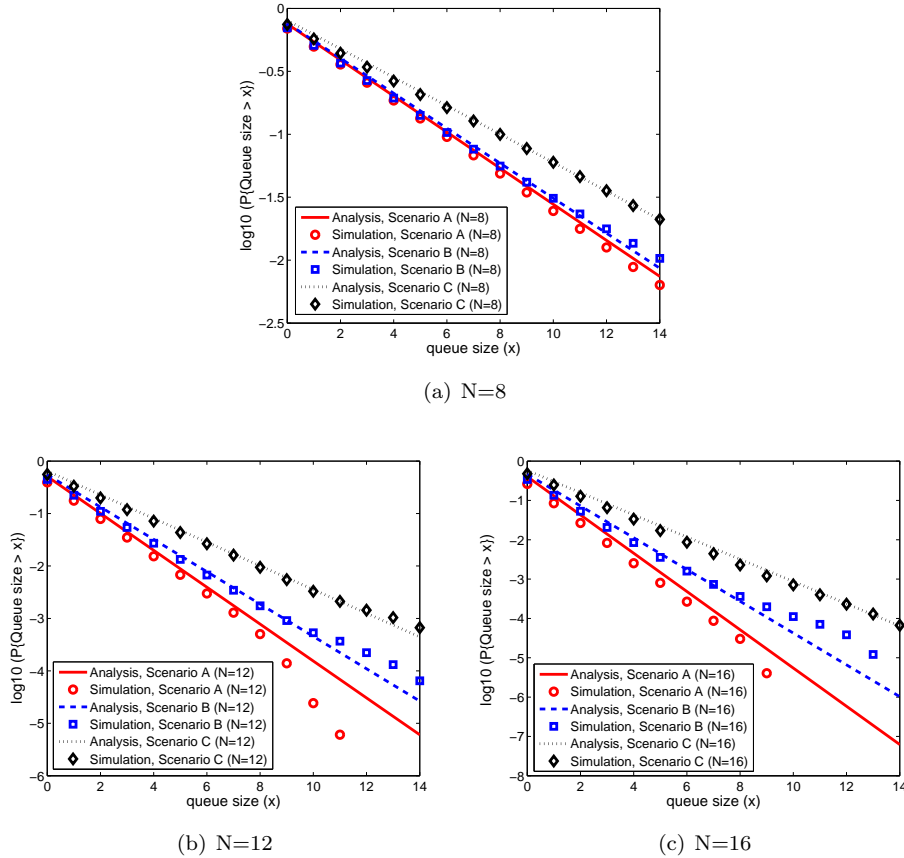


FIGURE 6. The queuing performance for  $N > sM$  ( $s = 0.7, M = 8$ )

**7.4. Queuing performance of the optimal access policy for  $N > sM$ .** We next investigate the queuing performance of the optimal channel access policies for three scenarios when  $N > sM$ . When  $N > sM$ , the optimal channel access policies in three scenarios are not equivalent. We use the same parameters for each wireless channel as given in Fig. 1, but the Poisson arrival rate is assumed to be 0.2. We fix  $M$  to be 8, change  $N$  from 8 to 16, and plot the resulting tail probabilities in Fig. 6. By comparing the figures in Fig. 6, when  $N$  increases, queuing performance in each scenario gets better and the difference in queuing performance for three scenarios becomes more significant. Moreover, from Fig. 6, we see that the knowledge on the channel status and conditions is beneficial to SUs in this case because queuing performance in Scenario A is better than that in Scenario B. Similarly, queuing performance in Scenario B is better than that in Scenario C. Hence, when we have enough channels, SUs had better know the information of channel status and conditions to get better queuing performance even for the optimal channel access policy.

**8. Conclusions.** In this paper, we considered a cognitive radio network where multiple secondary users contend to access wireless channels under Rayleigh fading.

We considered three scenarios according to the knowledge of the channel status and conditions. In each scenario, we consider a channel access policy where each secondary user stochastically determines whether to access an available wireless channel based on a given access probability. When a secondary user determines to access a channel, the secondary user selects one of available channels based on the information of the channel status and conditions.

To design the optimal access policy in each scenario from the queueing performance perspective, we analyzed the queueing performance of an arbitrary secondary user with help of the effective bandwidth theory. From our analysis, we obtained the optimal access probabilities that maximize queueing performance. We also showed that the optimal access policies with the optimal access probabilities in all scenarios are all equivalent under a certain condition. From numerical analysis we validated our analysis and investigated the fading effect on queueing performance.

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## Appendix

**The proof of Theorem 6.1.** Most of the proofs here are adaptations of those in Hwang and Roy [8]. However, for the self-containedness, we provide the proofs here. To prove Theorem 6.1, we first need the following lemma.

**Lemma 8.1.** For  $1 \leq k \leq N$ ,  $\phi_k^A(-\theta)$  is minimized for each  $\theta (> 0)$  when  $a_k = \min(\frac{k}{M(1-(1-s)^k)}, 1)$ .

*Proof.* For each  $k$ , let  $f_k(x)$  be defined by

$$f_k(x) := x \left(1 - \frac{1 - (1-s)^k}{k} x\right)^{M-1}, \quad 0 \leq x \leq 1, \quad M \geq 2.$$

By differentiating  $f_k(x)$ , we get

$$f'_k(x) = \left(1 - \frac{1 - (1-s)^k}{k} x\right)^{M-2} \left(1 - \frac{M(1 - (1-s)^k)}{k} x\right).$$

By checking the condition  $f'_k(x) \geq 0$ , we see that  $f_k(x)$  strictly increases for  $0 \leq x \leq \min(\frac{k}{M(1-(1-s)^k)}, 1)$  and strictly decreases for  $\min(\frac{k}{M(1-(1-s)^k)}, 1) < x \leq 1$ . Hence,  $f_k(x)$  is maximized when  $x = \min(\frac{k}{M(1-(1-s)^k)}, 1)$ . We next observe that

$$\phi_k^A(-\theta) = 1 + (1 - (1-s)^k) f_k(a_k) (e^{-\theta} - 1), \quad \theta > 0.$$

Since  $e^{-\theta} - 1 < 0$  for  $\theta > 0$ , the maximization of  $f_k(x)$  results in the minimization of  $\phi_k^A(-\theta)$ . Accordingly  $\phi_k^A(-\theta)$  is minimized when  $a_k = \min(\frac{k}{M(1-(1-s)^k)}, 1)$  for each  $\theta > 0$ .  $\square$

To prove Theorem 6.1, we need to show that for any APs  $\{a_1, a_2, \dots, a_N\}$  the following inequality is established

$$\xi_C^A(\theta; a_1, a_2, \dots, a_N) < \xi_C^A(\theta, a_1^*, a_2^*, \dots, a_N^*),$$

where  $\xi_C^A(\theta; a_1, a_2, \dots, a_N)$  is the EBF of the service capacity process in Scenario A when we use  $\{a_1, a_2, \dots, a_N\}$  and  $\xi_C^A(\theta; a_1^*, a_2^*, \dots, a_N^*)$  is the EBF of the service capacity process in Scenario A when we use  $\{a_1^*, a_2^*, \dots, a_N^*\}$ .

From Lemma 8.1 we observe that

$$\phi_1^A(-\theta; a_1) > \phi_1^A(-\theta; a_1^*),$$

where  $\phi_1^A(-\theta; a_1) = E[e^{\theta c_1(t; a_1)} | N(t) = 1]$  and  $c_1(t; a_1)$  is the number of successfully transmitted packets by the tagged SU at time slot  $t$  when the tagged SU uses the AP  $a_1$  for  $N(t) = 1$ . Then, along the lines of Theorem 1 of [8] (or by using the fact that the PF eigenvalue of an irreducible nonnegative matrix increases as any element in the matrix increases [12]) we can show that

$$\delta_C^A(-\theta; a_1, a_2, \dots, a_N) > \delta_C^A(-\theta; a_1^*, a_2, \dots, a_N),$$

where  $\delta_C^A(-\theta; a_1, a_2, \dots, a_N)$  is the PF eigenvalue of  $\Phi^A(\theta)R$  when we use  $\{a_1, \dots, a_N\}$ . Hence, we have that

$$\xi_C^A(\theta; a_1, a_2, \dots, a_N) < \xi_C^A(\theta; a_1^*, a_2, \dots, a_N).$$

Along the lines of Theorem 1 of [8], we can similarly show that

$$\xi_C^A(\theta; a_1, a_2, \dots, a_N) < \xi_C^A(\theta; a_1^*, a_2^*, \dots, a_N^*).$$

This implies that, when we use the values  $\{a_1^*, a_2^*, \dots, a_N^*\}$  of APs, the resulting EBF of the service capacity process is maximized for each  $\theta (> 0)$  and hence the queueing performance is optimized.

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