Design of a Fair Scheduler Exploiting Multiuser Diversity with Feedback Information Reduction

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Abstract—In this letter, we propose a new downlink fair scheduling scheme exploiting the multiuser diversity to enhance the transmission capacity. In the proposed scheme, only the MSs (mobile stations) whose normalized SNR (signal-to-noise ratio) values are larger than a given threshold feed back one-bit information to the BS (base station). As a result, while achieving the strict fairness, the proposed scheme can efficiently utilize the spectrum by reducing the considerable amount of the feedback information, compared to the proportional fair scheduling scheme where all the MSs feed back the normalized SNR values to the BS. Numerical studies show that the transmission capacity in the proposed scheme with a suitable value of the threshold is very close to that in the proportional fair scheduling scheme.

Index Terms—Multiuser Diversity, Normalized SNR, Fair Scheduling, Feedback Information Reduction

I. INTRODUCTION

Multiuser diversity comes from the fact that the wireless channel state processes of different users are usually independent for the same shared medium. To achieve the efficient use of the bandwidth, the multiuser diversity can be exploited in such a way that the scheduler in the BS (Base Station) selects the MS (Mobile Station) whose received SNR (Signal-to-Noise Ratio) is the best, and transmits packets to the selected MS. This scheduling scheme maximizes the (average) network capacity, but it is highly unfair when an MS has very disparate channel condition [1]. To solve this unfair problem, the proportional fair scheduling scheme has been proposed where the scheduler considers the normalized SNR values of MSs, defined by the received SNR values divided by the corresponding average received SNR values, and selects the MS whose normalized SNR value is the largest. This scheduling scheme provides a strict fairness among MSs because the normalized SNR values of all MSs at an arbitrary epoch are i.i.d. (independent and identically distributed) [1]. Here, the strict fairness means that the access probabilities of all MSs to the wireless channel are all equal.

When there are a lot of MSs in a network, the amount of feedback information, e.g., the Channel Quality Information (CQI), is significant and feedback bandwidth and the power get wasted [2]. So, reducing the feedback information without degrading the network performance significantly, has also been important, which is the main motivation of this study.

In this letter, we consider a wireless network consisting of a BS and K MSs. We assume that there is a scheduler in the BS, and focus on the downlink transmission between the BS and MSs. To design a fair scheduling scheme, we consider normalized SNR processes of MSs. In addition, to minimize the amount of feedback information we introduce a threshold and only the MSs with the normalized SNR values larger than the threshold are allowed to feed back one-bit information to the BS. In the subsequent sections we give the details of the proposed scheduling scheme and analyze it to see its performance. Through numerical studies we see that, if we select a suitable threshold value, then the proposed scheduler can achieve more than 97% of transmission capacity in the proportional fair scheduling scheme where the normalized SNR values are completely fed back to the BS. Hence, compared to the proportional fairness scheduling scheme, the proposed scheme can greatly reduce the feedback information at the little expense of the transmission capacity, while keeping the strict fairness.

II. SYSTEM MODEL AND ANALYSIS

In this letter, the downlink channel of MS is assumed to be described by \( r_i(t) = h_i(t)x(t) + n_i(t), \) \( i = 1, 2, \cdots, K, \) where \( x(t) \) is the transmitted signal in time slot \( t, \) \( r_i(t) \) is the received signal of MS \( i \) in time slot \( t, \) \( h_i(t) \) is the fading channel gain of MS \( i \) and \( n_i(t) \) is the zero mean complex Gaussian process with variance \( \sigma^2. \) We assume that the fading channel of MS \( i \) is according to the flat Rayleigh fading model, and that the received SNR \( \gamma_i(t) \) of MS \( i, \) defined by \( \gamma_i(t) = |h_i(t)|^2/\sigma^2, \) has average \( \bar{\gamma}_i. \) We further assume that \( \bar{\gamma}_i \) are different from MS to MS. For simplicity, we will drop the time index \( t \) from now on because we will consider the (average) transmission capacities of MSs.

A. Proposed Scheme

In the proposed scheme, a threshold value is given \( a \text{ priori} \) and only the MS whose normalized SNR value is larger than the threshold value is allowed to feed back its channel state to the BS at each time slot. We assume an error-free feedback channel with no delay in this letter.

In the proposed scheme, if an MS has the normalized SNR value larger than the threshold value, then it feeds back one-bit information which indicates its normalized SNR value is larger than the threshold value. The scheduler considers only the MSs who feed back and selects one of them randomly, if
any. If there are no MSs who feed back, then the scheduler considers all MSs and selects one of them randomly.

The merits of the proposed scheme are summarized as follows: First, it provides the strict fairness since it considers the normalized SNR values of all MSs at each epoch which are i.i.d. We will discuss it in detail in section II-B. Second, it reduces the amount of the feedback information in the network since only the MSs whose normalized SNR values are larger than the threshold value are allowed to feed back. In addition, since the proposed scheme uses only one-bit information for each MS, the amount of reduced information in the proposed scheme is significant compared with the proportional fair scheduling scheme. Note that from the view of the spectral efficiency, the reduction of the feedback information is highly desirable, especially, when the number of MSs is relatively large. So, the proposed scheme is effective if the transmission capacity degradation in the proposed scheme is not very significant and there are a large number of MSs in the network. We will check in section III if the transmission capacity degradation in the proposed scheme is significant or not.

It is worth while to mention the work by Gesbert and Alouini [3]. They considered homogeneous channel conditions for all MSs, and MSs whose received SNR value is greater than a given threshold are allowed to feed back. In their work, the BS selects the MS whose received SNR value is the greatest. Since the proposed scheme considers the normalized SNR values of MSs, not the received SNR values, we are allowed to design a strict fair scheduling scheme in the heterogeneous environment. In addition, since we select one of MSs who feed back randomly, we need one-bit information in the feedback, while the scheme in [3] needs the received SNR values of MSs in the feedback. These are the main differences between the proposed scheme and the scheme in [3].

B. Transmission Capacity Analysis

In this subsection, we analyze the transmission capacities of MSs in the proposed scheme. Let \( \gamma_{th} \) denote the threshold value in the proposed scheme. Since the wireless fading channel is Rayleigh, the probability density function (p.d.f.) \( f_i(x) \) of the normalized SNR value \( s_i = \gamma_i/\bar{\gamma}_i \) for MS \( i \) is given by \( f_i(x) = e^{-x}, x \geq 0 \) for all \( i = 1, 2, \ldots, K \). Then the probability that \( s_i \) is above the threshold \( \gamma_{th} \), denoted by \( \pi_g \), is given by \( \pi_g = \int_0^{\infty} f_i(x) \, dx = e^{-\gamma_{th}} \). In addition, the probability that \( s_i \) is below the threshold \( \gamma_{th} \), denoted by \( \pi_b \), is given by \( \pi_b = 1 - \pi_g = 1 - e^{-\gamma_{th}} \). Here, the subscripts ‘g’ and ‘b’ stand for ‘good’ and ‘bad’, respectively.

For the analysis of the transmission capacity of MS \( i \), we tag MS \( i \) and all the other MSs are called untagged MSs. Since normalized SNR values are i.i.d., the probability \( \delta_k \) that the number of untagged MSs whose normalized SNR values are above \( \gamma_{th} \) is \( k \), is given by \( \delta_k = (K-1) \pi_g \pi_b^{K-1-k}, 0 \leq k \leq K-1 \).

Next, given that all MSs have normalized SNR values less than \( \gamma_{th} \), the conditional p.d.f. \( f_i^{(l)}(x) \) of \( s_i \) is given by

\[
f_i^{(l)}(x) = \frac{e^{-x}}{1 - e^{-\gamma_{th}}}, \quad 0 \leq x < \gamma_{th}.
\]  

Note that, since all \( s_j \)'s \( (j = 1, \ldots, K) \) are independent, \( f_i^{(l)} \) is in fact the conditional p.d.f. of \( s_i \), given that \( s_i \leq \gamma_{th} \).

We are now ready to analyze the transmission capacity of MS \( i \). In the proposed scheme, the transmission capacity of MS \( i \) is denoted by \( C_i \) and given by

\[
C_i = \sum_{k=0}^{K-1} \delta_k \pi_g \frac{1}{k+1} \int_{\gamma_{th}}^{\infty} \log_2(1 + \gamma_i x) \frac{e^{-x}}{e^{-\gamma_{th}}} \, dx
+ \delta_0 \pi_b \frac{1}{K} \int_0^{\gamma_{th}} \log_2(1 + \gamma_i x) f_i^{(l)}(x) \, dx. \tag{2}
\]

Here, the first term on the right hand side is the transmission capacity of MS \( i \) when at least one MS including MS \( i \) has normalized SNR value above \( \gamma_{th} \) and MS \( i \) is selected for transmission by the random selection strategy of the scheduler. The second term is the transmission capacity of MS \( i \) when \( K \) MSs in the network have all normalized SNR values less than \( \gamma_{th} \) and MS \( i \) is selected for transmission by the random selection strategy of the scheduler. After some manipulations we get the following theorem whose proof is given in Appendix.

**Theorem 1:** The transmission capacity of MS \( i \) in the proposed scheme is given by

\[
C_i = \frac{1}{\ln 2} \sum_{k=0}^{K-1} \delta_k \pi_g \left( \frac{1}{k+1} \left( 1 + \frac{\gamma_{th}}{\gamma_i} \right) \log_2 \left( 1 + \frac{1}{\gamma_i} \right) - E_1 \left( \frac{1}{\gamma_i} \right) \right)
+ \delta_0 \pi_b \frac{1}{K} \left( \frac{1}{\gamma_i} \right) - E_1 \left( \frac{1}{\gamma_i} \right) \right) \}
\]

where \( E_1(\cdot) \) is the exponential integral defined by \( E_1(x) = \int_1^\infty \frac{e^{-x t}}{t} \, dt \).

With respect to the fairness of the proposed scheme, for this purpose we use the self-fairness \( F_i \) of MS \( i \) introduced in [1] and defined by \( F_i = \frac{-\log(P_i)}{\log K} \) where \( P_i \) is the amount of time for transmission allocated to MS \( i \), or, equivalently, the access probability, and \( K \) is the number of MSs in the network. Using the same argument in the derivation of equation (2), the access probability, \( P_i \), of MS \( i \) is computed as \( P_i = \sum_{k=0}^{K-1} \delta_k \pi_g \pi_b^{K-1-k} + \delta_0 \pi_b \frac{1}{K} \), from which we see that \( F_1 = F_2 = \cdots = F_K \) and accordingly the strict fairness of the proposed scheduling scheme is shown.

For comparison, we compute the transmission capacity \( C_i^* \) of MS \( i \) in the proportional fair scheduling scheme based on the normalized SNR values. Noting that

\[
P\{\max(s_1, \ldots, s_K) = s_i, s_i \leq x\} = \int_0^x (1 - e^{-t})^{K-1} e^{-t} \, dt,
\]

the pdf \( f(x) \) of the maximum normalized SNR value \( s_i \) is given by \( f(x) = (1 - e^{-x})^{K-1} e^{-x}, x \geq 0 \). Then, the transmission capacity \( C_i^* \) of MS \( i \) is computed by

\[
C_i^* = \int_0^{\infty} \log_2(1 + \gamma_i x) f(x) \, dx
= \frac{1}{\ln 2} \sum_{k=0}^{K-1} \left( \frac{K-1}{k} \right) \frac{(-k)^{k-1}}{k+1} e^{\gamma_i} E_1 \left( \frac{1}{\gamma_i} \right). \tag{3}
\]
In this letter, we proposed a new downlink fair scheduling scheme exploiting the multiuser diversity. In the proposed scheme, only the MSs with normalized SNR values larger than a given threshold are allowed to feed back one-bit information, which can significantly reduce the feedback information. Through mathematical analysis and numerical studies we see that, compared to the proportional fairness scheduling, the proposed scheme with a suitable threshold value can greatly reduce the feedback information at the little expense of the transmission capacity, while keeping the strict fairness.

### Appendix

In the appendix, we first provide two integral formulas which are useful to prove Theorem 1.

**Lemma 1:** For \(a, b > 0\),

\[
\int_a^\infty e^{-bx} \ln x \, dx = \frac{1}{b} e^{-ab} \ln a + \frac{1}{b} E_1(ab),
\]

\[
\int_a^\infty e^{-bx} \ln x \, dx = -\frac{1}{b} e^{-ab} \ln a + \frac{1}{b} (E_1(b) - E_1(ab)).
\]

**Proof:** By integration by parts, our lemma follows.

#### A. Proof of Theorem 1

We first compute the first term on the right hand of equation (2) as follows:

\[
\sum_{k=0}^{K-1} \delta_k \pi_2 \int_{\gamma_i}^{\infty} \frac{1}{k+1} \ln(1 + \gamma_i x) e^{-\gamma_i x} \, dx
\]

\[
= \sum_{k=0}^{K-1} \delta_k \pi_2 (k+1) \ln 2 \int_{\gamma_i}^{\infty} (1 + \gamma_i x) e^{-\gamma_i x} \, dx
\]

\[
= \sum_{k=0}^{K-1} \delta_k \pi_2 (k+1) \ln 2 \frac{1}{\gamma_i} \int_{1+\gamma_i \gamma_i}^{\infty} e^{-\frac{y}{\gamma_i}} \ln y \, dy
\]

by putting \(y = 1 + \gamma_i x\)

\[
= \frac{1}{\ln 2} \sum_{k=0}^{K-1} \delta_k \pi_2 (k+1) \left( e^{-\gamma_i \gamma_i} \ln(1 + \gamma_i \gamma_i) + e^{-\frac{1}{\gamma_i}} E_1 \left( 1 + \frac{1}{\gamma_i} \gamma_i \right) \right)
\]

by using Lemma 1.

Similarly, using equation (1) and Lemma 1, we can compute the second term on the right hand side of equation (2), which completes the proof.

### References

